

5. (a)  $y = \tan^{-1} x$

Assuming the derivative of  $\tan x$ , prove that

$$\frac{dy}{dx} = \frac{1}{1+x^2} \quad (3)$$

$$f(x) = x \tan^{-1} 4x$$

(b) Show that

$$\int f(x) dx = Ax^2 \tan^{-1} 4x + Bx + C \tan^{-1} 4x + k$$

where  $k$  is an arbitrary constant and  $A$ ,  $B$  and  $C$  are constants to be determined.

(5)

(c) Hence find, in exact form, the mean value of  $f(x)$  over the interval  $\left[0, \frac{\sqrt{3}}{4}\right]$

(2)