7.

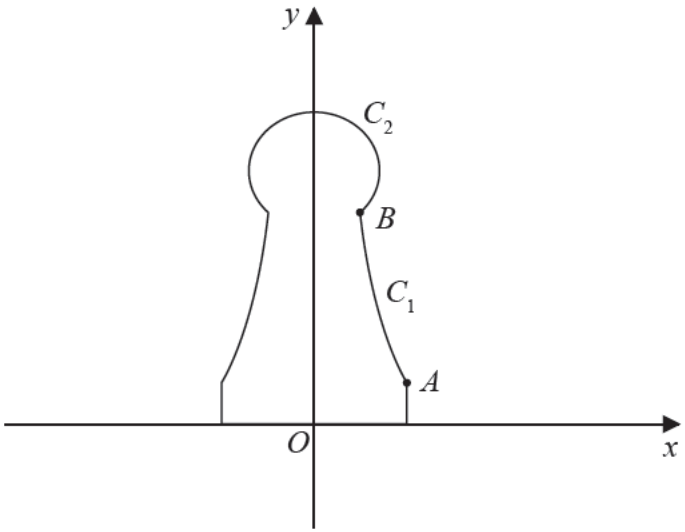

Figure 1

A student wants to make plastic chess pieces using a 3D printer. Figure 1 shows the central vertical cross-section of the student's design for one chess piece. The plastic chess piece is formed by rotating the region bounded by the y-axis, the x-axis, the line with equation $x = 1$, the curve C_1 and the curve C_2 through 360° about the y-axis.

The point A has coordinates (1, 0.5) and the point B has coordinates (0.5, 2.5) where the units are centimetres.

The curve C_1 is modelled by the equation

$$x = \frac{a}{y + b} \qquad 0.5 \leqslant y \leqslant 2.5$$

(a) Determine the value of a and the value of b according to the model.

(2)

The curve C_2 is modelled to be an arc of the circle with centre (0, 3).

(b) Use calculus to determine the volume of plastic required to make the chess piece according to the model.

(9)