Question	Scheme	Marks	AOs
4(a)	A complete attempt to find the sum of the cubes of the first *n* odd numbers using three of the standard summation formulae. Attempts to find $\sum(2r+1)^3$ or $\sum(2r-1)^3$ by expanding and using summation formulae	M1	3.1a
	$\sum_{r=1}^{n}(2r-1)^3=\sum_{r=1}^{n}(8r^3-12r^2+6r-1)=8\sum_{r=1}^{n}r^3-12\sum_{r=1}^{n}r^2+6\sum_{r=1}^{n}r-\sum_{r=1}^{n}1$ or $\sum_{r=0}^{n-1}(2r+1)^3=\sum_{r=0}^{n-1}(8r^3+12r^2+6r+1)=8\sum_{r=0}^{n-1}r^3+12\sum_{r=0}^{n-1}r^2+6\sum_{r=0}^{n-1}r+\sum_{r=0}^{n-1}1$	M1	1.1b
	$=8\frac{n^2}{4}(n+1)^2-12\frac{n}{6}(n+1)(2n+1)+6\frac{n}{2}(n+1)-n$ or $=8\frac{(n-1)^2}{4}(n)^2+12\frac{(n-1)}{6}(n)(2n-1)+6\frac{(n-1)}{2}(n)+n$	M1 A1	1.1b 1.1b
	Multiplies out to achieve a correct intermediate line for example $n \quad n+1 \quad 2n^2-2n+1 \quad -n=2n^4-2n^3+n^2+2n^3-2n^2+n-n$ $2n^4+4n^3+2n^2-4n^3-6n^2-2n+3n^2+3n-n$ leading to $=n^2(2n^2-1)$ cso *	A1 *	2.1
		(5)	
(b)	$\sum_{r=n}^{n+9}(2r-1)^3=\sum_{r=1}^{n+9}(2r-1)^3-\sum_{r=1}^{n-1}(2r-1)^3$ $=(n+9)^2\left(2(n+9)^2-1\right)-(n-1)^2\left(2(n-1)^2-1\right)=99800$ or $\sum_{r=n+1}^{n+10}(2r-1)^3=\sum_{r=1}^{n+10}(2r-1)^3-\sum_{r=1}^{n}(2r-1)^3$ $=(n+10)^2\left(2(n+10)^2-1\right)-(n)^2\left(2n^2-1\right)=99800$ or $\sum_{r=n-9}^{n}(2r-1)^3=\sum_{r=1}^{n}(2r-1)^3-\sum_{r=1}^{n-10}(2r-1)^3$ $=(n)^2\left(2(n)^2-1\right)-(n-10)^2\left(2(n-10)^2-1\right)=99800$	M1	3.1a
	$80n^3+960n^2+5820n-86760=0$ or $80n^3+1200n^2+7980n-79900=0$ or $80n^3-1200n^2+7980n-119700=0$	A1	1.1b
	Solves cubic equation	dM1	1.1b

	Achieves $n=6$ and the smallest number as 11 or Achieves $n=5$ and the smallest number as 11 or Achieves $n=15$ and the smallest number as 11	A1	2.3
		(4)	

(9 marks)

Notes:

(a)

M1: A complete attempt to find the sum of the cubes of *n* odd numbers using three of the standard summation formulae.

M1: Expands $\sum_{r=1}^{n}(2r-1)^3$ or $\sum_{r=0}^{n-1}(2r+1)^3$ and splits into fours appropriate sums.

M1: Applies the result for at least three summations $\sum_{r=0}^{n-1}r^3, \sum_{r=0}^{n-1}r^2, \sum_{r=0}^{n-1}r$ and $\sum_{r=0}^{n-1}1$ or $\sum_{r=1}^{n}r^3, \sum_{r=1}^{n}r^2, \sum_{r=1}^{n}r$ and $\sum_{r=1}^{n}1$ as appropriate to their expansion provided that there is an attempt at cubing some values.

A1: Correct unsimplified expression.

A1 *: Multiplies out to achieve a correct intermediate expression which clearly leads to the correct expression. cso

Special case: If uses $\sum_{r=1}^{n}(2r+1)^3$ leading to $=8\frac{n^2}{4}(n+1)^2+12\frac{n}{6}(n+1)(2n+1)+6\frac{n}{2}(n+1)+n$ max score is M1 M0 M1 A1 A0

(b)

M1: Uses the answer to part (a) to find the **s**um of the cubes of the first $N+10$ odd numbers minus the sum of the first N odd numbers and sets equal to 99800 or equivalent.

A1: Correct simplified cubic equation.

dM1: Uses their calculator to solve their cubic equation, dependent on previous method mark.

A1: cao