

| Question    | Scheme   | Marks | AOs  |
|-------------|--|-------|------|
| <b>5(a)</b> | $\frac{dy}{dx} = \frac{-\lambda}{\sqrt{1-\beta x^2}}$ where $\lambda > 0$ and $\beta > 0$ and $\beta \neq 1$<br>Alternatively $2 \cos y = x \Rightarrow \frac{dx}{dy} = \alpha \sin y \Rightarrow \frac{dy}{dx} = \frac{1}{\alpha \sin y}$   | M1    | 1.1b |
|             | $\frac{dy}{dx} = \frac{-\frac{1}{2}}{\sqrt{1-\frac{1}{4}x^2}}$ or $\frac{dy}{dx} = \frac{-1}{2\sqrt{1-\frac{1}{4}x^2}}$ o.e.<br>or $\frac{dy}{dx} = -\frac{1}{2 \sin y}$ or  | A1    | 1.1b |
|             | States that $\frac{dy}{dx} \neq 0$ therefore $C$ has no stationary points.<br>Tries to solve $\frac{dy}{dx} = 0$ and ends up with a contradiction e.g. $-1 = 0$<br>therefore $C$ has no stationary points.<br>As $\operatorname{cosec} y > 1$ therefore $C$ has no stationary points.                  | A1    | 2.4  |
|             |  | (3)   |      |
| <b>(b)</b>  | $\frac{dy}{dx} = \frac{-1}{2\sqrt{1-\frac{1}{4}x^2}} = \left\{ -\frac{1}{\sqrt{3}} \right\}$   | M1    | 1.1b |
|             | Normal gradient = $-\frac{1}{m}$ and $y - \frac{\pi}{3} = m_n(x-1)$<br>Alternatively $\frac{\pi}{3} = m_n(1) + c \Rightarrow c = \dots \left\{ \frac{\pi}{3} - \sqrt{3} \right\}$ and then $y = m_n x + c$   | M1    | 1.1b |
|             | $y = 0 \Rightarrow 0 - \frac{\pi}{3} = \sqrt{3}(x_A - 1) \Rightarrow x_A = \dots \left\{ 1 - \frac{\pi}{3\sqrt{3}} \text{ or } 1 - \frac{\pi\sqrt{3}}{9} \right\}$<br>and<br>$x = 0 \Rightarrow y_B - \frac{\pi}{3} = \sqrt{3}(0-1) \Rightarrow y_B = \dots \left\{ \frac{\pi}{3} - \sqrt{3} \right\}$ | M1    | 3.1a |
|             | $\text{Area} = \frac{1}{2} \times x_A \times -y_B = \frac{1}{2} \left( 1 - \frac{\pi}{3\sqrt{3}} \right) \left( \sqrt{3} - \frac{\pi}{3} \right)$  | M1    | 1.1b |
|             | $\text{Area} = \frac{1}{54} (27\sqrt{3} - 18\pi + \sqrt{3}\pi^2) \quad (p = 27, q = -18, r = 1)$   | A1    | 2.1  |
|             |  | (5)   |      |

(8 marks)

**Notes:**

**(a)**  
**M1:** Finds the correct form for  $\frac{dy}{dx}$

**A1:** Correct  $\frac{dy}{dx}$

**A1:** States or shows that  $\frac{dy}{dx} \neq 0$  and draws the required conclusion. This mark can be scored as long as the M mark has been awarded.

**(b)**

**M1:** Substitutes  $x = 1$  into their  $\frac{dy}{dx}$

**M1:** Finds the normal gradient and finds the equation of the normal using  $y - \frac{\pi}{3} = m_n(x - 1)$

**M1:** Finds where their normal cuts the  $x$ -axis and the  $y$ -axis.

**M1:** Finds the area of the triangle  $OAB = \frac{1}{2} \times x_A \times -y_B$ .

**A1:** Correct area

Special case: If finds the tangent to the curve, the  $x$  and  $y$  intercepts and the area of the triangle max score M1 M0 M1 M0 A0

Note common error

$\frac{dy}{dx} = \frac{-1}{\sqrt{1 - \frac{1}{4}x^2}}$  In part (b) this leads to  $\frac{dy}{dx} = \frac{-2}{\sqrt{3}}$  leading to normal gradient  $\frac{\sqrt{3}}{2}$  and

$y = \frac{\sqrt{3}}{2}x - \frac{\sqrt{3}}{2} + \frac{\pi}{3}$  and  $\left(0, \frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)$  and  $\left(1 - \frac{2\pi}{3\sqrt{3}}, 0\right)$  therefore area =  $\frac{1}{2} \left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right) \left(\frac{2\pi}{3\sqrt{3}} - 1\right)$

This can score M1 M1 M1 M1 A0