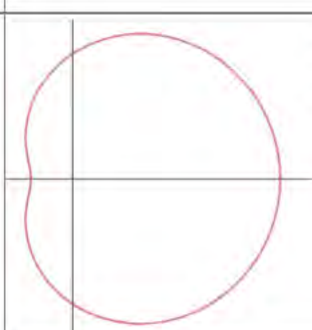
Question	Scheme	Marks	AOs
6(a)	$x = r\cos\theta = a(p+2\cos\theta)\cos\theta$ Leading to $\frac{\mathrm{d}x}{\mathrm{d}\theta} = \alpha\sin\theta\cos\theta + \beta\sin\theta(p+2\cos\theta)$ or $\frac{\mathrm{d}x}{\mathrm{d}\theta} = \alpha\sin\theta\cos\theta + \beta\sin\theta$ or $x = a(p\cos\theta + 2\cos^2\theta) = a(\cos 2\theta + p\cos\theta + 1)$ leading to $\frac{\mathrm{d}x}{\mathrm{d}\theta} = \alpha\sin 2\theta + \beta\sin\theta$	M1	3.1a
	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = a\left[-2\sin\theta\cos\theta - \sin\theta(p+2\cos\theta)\right]$ or $\frac{\mathrm{d}x}{\mathrm{d}\theta} = -4a\sin\theta\cos\theta - ap\sin\theta$ or $\frac{\mathrm{d}x}{\mathrm{d}\theta} = -2a\sin 2\theta - ap\sin\theta$	A1	1.1b
	$a\left[-2\sin\theta\cos\theta - \sin\theta(p+2\cos\theta)\right] = 0$ $\pm a(4\sin\theta\cos\theta + p\sin\theta) = 0$ $a\sin\theta(4\cos\theta + p) = 0$ Either $\sin\theta = 0$ or $\cos\theta = -\frac{p}{4}$	M1	3.1a
	$\sin\theta = 0$ implies 2 solutions (tangents which are perpendicular to the initial line) e.g. $\theta = 0, \pi$	B1	2.2a
	Therefore two solutions to $\cos\theta = -\frac{p}{4}$ are required $-\frac{p}{4} > -1 \Rightarrow p < 4$ as p is a positive constant $2 < p < 4$ *	A1*	2.4
		(5)	
(b)	 Correct shape and position. Condone cusp	B1	2.2a
		(1)	
(c)	Area = $2\times\frac{1}{2}\int_0^{\pi}\left[20(3+2\cos\theta)\right]^2\,\mathrm{d}\theta = 400\int_0^{\pi}(9+12\cos\theta+4\cos^2\theta)\,\mathrm{d}\theta$ or $=\int_0^{\pi}(3600+4800\cos\theta+1600\cos^2\theta)\,\mathrm{d}\theta$	M1	3.4

	or $\frac{1}{2}\int_0^{2\pi}\left[20(3+2\cos\theta)\right]^2\,\mathrm{d}\theta = 200\int_0^{2\pi}\left(9+12\cos\theta+4\cos^2\theta\right)\mathrm{d}\theta$ or $=\int_0^{2\pi}\left(1800+2400\cos\theta+800\cos^2\theta\right)\mathrm{d}\theta$		
	$\cos^2\theta=\frac{1}{2}+\frac{1}{2}\cos 2\theta \Rightarrow$ $A=...\int\left(9+12\cos\theta+2+2\cos 2\theta\right)\mathrm{d}\theta=\alpha\theta\pm\beta\sin\theta\pm\lambda\sin 2\theta$	M1	3.1a
	$=400\left[11\theta+12\sin\theta+\sin 2\theta\right]$ or $=200\left[11\theta+12\sin\theta+\sin 2\theta\right]$	A1	1.1b
	Using limits $\theta=0$ and $\theta=\pi$ or $\theta=0$ and $\theta=2\pi$ as appropriate and subtracts the correct way round provided there is an attempt at integration $=400\left[11\pi-0\right]=4400\pi=13823.0\left(\text{cm}^2\right)$ or $=200\left[11(2\pi)-0\right]=4400\pi=13823.0\left(\text{cm}^2\right)$	M1	1.1b
	Volume $=$ area $\times 90 = 396\,000\pi = 1\,244\,070.691\left(\text{cm}^3\right)$	M1	3.4
	time = $\frac{1\,244\,070.691}{50\,000}=...$ or volume = 1244 litres therefore time = $\frac{1244}{50}=...$	M1	2.2b
	25 (minutes)	A1	3.2a
		(7)	
(d)	For example Polar equation is not likely to be accurate. Some comment that the sides will not be smooth and draws an appropriate conclusion. The hole may not be uniform depth The pond may leak/ ground may absorb some water	B1	3.5b
		(1)	
			(14 marks)

Notes:

(a)

M1: Complete method to find the correct form for $\frac{\mathrm{d}x}{\mathrm{d}\theta}$

A1: Correct $\frac{\mathrm{d}x}{\mathrm{d}\theta}$

M1: Sets $\frac{\mathrm{d}x}{\mathrm{d}\theta}=0$ and factorises to find values for either $\sin\theta$ or $\cos\theta$.

B1: Deduces that as $\sin\theta=0$ this provides two tangents. This can be implied by 2 values for θ

A1*: Concludes that as $\cos\theta=-\frac{p}{4}>-1\Rightarrow p<4$ and p is a positive constant $\therefore\ 0<p<4$

(b) **B1:** Correct shape and position.
(c) **M1:** Uses the model to find the area of the cross section $2\times\frac{1}{2}\int_0^{\pi}\left[20\left(3+2\cos\theta\right)\right]^2\,\mathrm{d}\theta$ or $\frac{1}{2}\int_0^{2\pi}\left[20\left(3+2\cos\theta\right)\right]^2\,\mathrm{d}\theta$ **M1:** Uses the identity $\cos 2\theta = 2\cos^2\theta - 1$ to integrate to the required form. **A1:** Correct integration. **M1:** Uses limits $\theta = 0$ and $\theta = \pi$ or $\theta = 0$ and $\theta = 2\pi$ as appropriate and subtracts the correct way around provided there is an attempt at integration. Note if first M1 is not awarded for incorrect limits then award this mark for their limits used. **M1:** Multiplies their area by 90 (cm). **M1**: Divides their volume by 50000 **A1:** 25 (minutes)
(d) **B1:** See scheme for examples. Any reference to the flow of water is B0