Question	Scheme		Marks	AOs
7(a)	Using $\text{arsinh}\,\alpha = \frac{1}{2}\ln 3$ $\alpha = \dfrac{e^{\frac{1}{2}\ln 3} - e^{-\frac{1}{2}\ln 3}}{2}$	$\ln\left(\alpha + \sqrt{\alpha^2+1}\right) = \frac{1}{2}\ln 3$	B1	1.2
	$\alpha = \dfrac{\sqrt{3} - \frac{1}{\sqrt{3}}}{2} \Rightarrow \alpha = \ldots$	$\alpha + \sqrt{\alpha^2+1} = \sqrt{3}$ $\sqrt{\alpha^2+1} = \sqrt{3} - \alpha$ $\alpha^2 + 1 = 3 - 2\sqrt{3}\alpha + \alpha^2 \Rightarrow \alpha = \ldots$	M1	1.1b
	$\alpha = \dfrac{\sqrt{3}}{3}$ or $\dfrac{1}{\sqrt{3}}$		A1	2.2a
			(3)	
(b)	Volume = $\pi\int_0^{\frac{1}{2}\ln 3} \sinh^2 y\, dy$		B1	2.5
	$\{\pi\}\int\left(\dfrac{e^y - e^{-y}}{2}\right)^2 dy = \{\pi\}\int\left(\dfrac{e^{2y} - 2 + e^{-2y}}{4}\right) dy$ or $\{\pi\}\int \frac{1}{2}\cosh 2y - \frac{1}{2}\, dy$		M1	3.1a
	$\frac{1}{4}\left(\frac{1}{2}e^{2y} - 2y - \frac{1}{2}e^{-2y}\right)$ or $\frac{1}{4}\sinh 2y - \frac{1}{2}y$		dM1 A1	1.1b 1.1b
	Use limits $y = 0$ and $y = \frac{1}{2}\ln 3$ and subtracts the correct way round		M1	1.1b
	$\frac{\pi}{4}\left(\frac{4}{3} - \ln 3\right)$ or exact equivalent		A1	1.1b
			(6)	
			(9 marks)	

Notes:

(a)

B1: Recalls the definition for $\sinh\left(\frac{1}{2}\ln 3\right)$ or forms an equation for arcsinh x

M1: Uses logarithms to find a value for α or forms and solves a correct equation without log

A1: Deduces the correct exact value for α

Note using the result

$$\ln\left(\frac{1}{\sqrt{3}} + \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + 1}\right) = \ln\left(\frac{1}{\sqrt{3}} + \sqrt{\frac{4}{3}}\right) = \ln\sqrt{3} = \frac{1}{2}\ln 3 \text{ therefore } \text{arsinh}\left(\frac{1}{\sqrt{3}}\right) = \frac{1}{2}\ln 3$$

B1 for substituting in α into arcsinh$x$, M1 for rearranging to show $\frac{1}{2}\ln 3$, A1 for conclusion
(b) **B1:** Correct expression for the volume $\pi\int_0^{\frac{1}{2}\ln 3}\sinh^2 y\,\mathrm{d}y$ requires integration signs, dy and correct limits. **M1:** Uses the exponential formula for sinh y or the identity $\cosh 2y = \pm 1 \pm 2\sinh^2 y$ to write in a form which can be integrated at least one term **dM1:** Dependent of previous method mark, integrates. **A1:** Correct integration. **M1:** Correct use of the limits $y = 0$ and $y = \frac{1}{2}\ln 3$ **A1**: Correct exact volume.