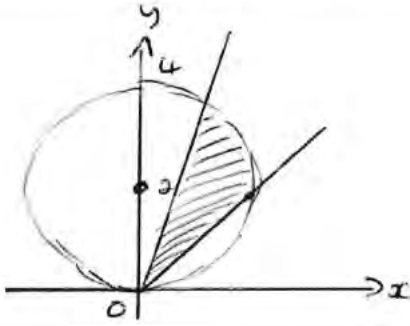


Question	Scheme	Marks	AOs
8(i)	$ z  = \sqrt{6^2 + 6^2} = \dots 6\sqrt{2}$ or $\sqrt{72}$ and $\arg z = \tan^{-1}\left(\frac{6}{6}\right) = \dots \left[\frac{\pi}{4}\right]$ Can be implied by $r = 6\sqrt{2}e^{4i}$	M1 A1	3.1a 1.1b
	Adding multiples of $\frac{2\pi}{5}$ to their argument $z = 6\sqrt{2}e^{4i} \times e^{\frac{2\pi k i}{5}}$ or $z = 6\sqrt{2} \left[ \cos\left(\frac{\pi}{4} + \frac{2\pi k}{5}\right) + i \sin\left(\frac{\pi}{4} + \frac{2\pi k}{5}\right) \right]$	M1	1.1b
	$z = re^{(\theta + \frac{2\pi}{5})i}, re^{(\theta + \frac{4\pi}{5})i}, re^{(\theta + \frac{6\pi}{5})i}, re^{(\theta + \frac{8\pi}{5})i}$ o.e. or $z = re^{(\theta + \frac{2\pi}{5})i}, re^{(\theta - \frac{2\pi}{5})i}, re^{(\theta - \frac{6\pi}{5})i}, re^{(\theta - \frac{8\pi}{5})i}$ o.e.	A1ft	1.1b
	$z = 6\sqrt{2}e^{\frac{13\pi i}{20}}, 6\sqrt{2}e^{\frac{21\pi i}{20}}, 6\sqrt{2}e^{\frac{29\pi i}{20}}, 6\sqrt{2}e^{\frac{37\pi i}{20}}$ o.e. or $z = 6\sqrt{2}e^{\frac{13\pi i}{20}}, 6\sqrt{2}e^{-\frac{19\pi i}{20}}, 6\sqrt{2}e^{-\frac{11\pi i}{20}}, 6\sqrt{2}e^{-\frac{3\pi i}{20}}$ o.e.	A1	1.1b
		(5)	
(ii)(a)	Circle centre (0, 2) and radius 2 or with the point on the origin	B1	1.1b
	Fully correct 	B1	1.1b
		(2)	
(ii)(b)	$\text{area} = \frac{1}{2} \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} 4 \sin^2 \theta \, d\theta$ or $\text{area} = \frac{1}{2} \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \alpha \sin^2 \theta \, d\theta$	M1	3.1a
	Uses $\sin^2 \theta = \frac{1}{2} - \frac{1}{2} \cos 2\theta$ and integrates to the form $A\theta + B \sin 2\theta$ $\text{area} = 8 \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sin^2 \theta \, d\theta = 4 \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} 1 - \cos 2\theta \, d\theta = 4\theta - 2 \sin 2\theta$	M1	3.1a
	Uses the limits of $\frac{\pi}{4}$ and $\frac{\pi}{3}$ and subtracts the correct way around $\left[ 4\left(\frac{\pi}{3}\right) - 2 \sin\left(\frac{2\pi}{3}\right) \right] - \left[ 4\left(\frac{\pi}{4}\right) - 2 \sin\left(\frac{2\pi}{4}\right) \right]$	M1	1.1b

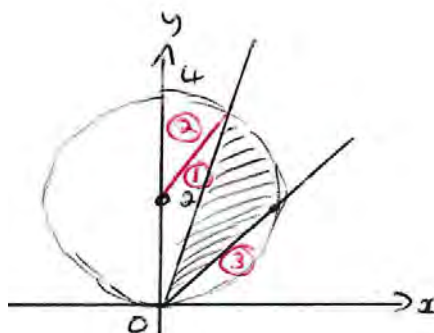
$$\text{Area} = \frac{\pi}{3} - \sqrt{3} + 2$$

A1

1.1b

(4)

**Alternative**



Finds either the areas 1 or 2

$$\text{Area 1} = \frac{1}{2} \times 2^2 \times \sin\left(\frac{2\pi}{3}\right) \{ = \sqrt{3} \}$$

$$\text{Area 2} = \frac{1}{2} \times 2^2 \times \frac{\pi}{3} \{ = \frac{2\pi}{3} \}$$

M1

1.1b

A complete method to find area 3

$$\text{Area 3} = \frac{1}{4} \pi \times 2^2 - \frac{1}{2} \times 2^2 \{ = \pi - 2 \}$$

M1

3.1a

A complete method to find the required area

$$\begin{aligned} \text{Shaded area} &= \text{Area of semi circle} - \text{area 1} - \text{area 2} - \text{area 3} \\ &= \left[ \frac{1}{2} \pi \times 2^2 \right] - \left[ \frac{1}{2} \times 2^2 \times \sin\left(\frac{2\pi}{3}\right) \right] - \left[ \frac{1}{2} \times 2^2 \times \frac{\pi}{3} \right] - \left[ \frac{1}{4} \pi \times 2^2 - \frac{1}{2} \times 2^2 \right] \\ &= 2\pi - \sqrt{3} - \frac{2\pi}{3} - (\pi - 2) \end{aligned}$$

Or

$$\begin{aligned} \text{Shaded area} &= \text{Area of sector} - \text{area 1} - \text{area 3} \\ &= \left[ \frac{1}{2} \times 4 \times \left(\frac{2\pi}{3}\right) \right] - \left[ \frac{1}{2} \times 2^2 \times \sin\left(\frac{2\pi}{3}\right) \right] - \left[ \frac{1}{4} \pi \times 2^2 - \frac{1}{2} \times 2^2 \right] \\ &= \frac{4\pi}{3} - \sqrt{3} - (\pi - 2) \end{aligned}$$

M1

3.1a

$$\text{Area} = \frac{\pi}{3} - \sqrt{3} + 2$$

A1

1.1b

(4)

(11 marks)

**Notes:**


- (i) **M1:** Finds the modulus and argument of  $z$
- A1:** Correct modulus and argument of  $z$

**M1:** Uses a correct method to find to all the other 4 vertices of the pentagon. Must be doing the equivalent of adding/ subtracting multiples of  $\frac{2\pi}{5}$  to the argument.

**A1ft:** All 4 vertices following through on their modulus and argument. Does not need to be simplified for this mark.

**A1:** All 4 vertices correct in the required form

**(ii)(a)**

**B1:** Circle centre (0, 2) and radius 2 or  with the vertex on the origin.

**B1:** Fully correct region shaded.

**(ii) (b)**

**M1:** Writes the required area using polar coordinates

**M1:** Uses  $\sin^2 \theta = \frac{1}{2} - \frac{1}{2} \cos 2\theta$  and integrates to the form  $A\theta + B \sin 2\theta$

**M1:** Uses the limits of  $\frac{\pi}{4}$  and  $\frac{\pi}{3}$  and subtracts the correct way around. Must be some attempt at

area =  $\frac{1}{2} \int \alpha \sin^2 \theta \, d\theta$  and integration.

**A1:** Correct exact area =  $\frac{\pi}{3} - \sqrt{3} + 2$

**Alternative**

**M1:** Finds either area 1 or area 2

**M1:** A complete method to find the area 3

**M1:** A complete method to find the required area = Area of semi circle – area 1 – area 2 – area 3 or  
= Area of sector – area 1 – area 3

**A1:** Correct exact area =  $\frac{\pi}{3} - \sqrt{3} + 2$