Question	Scheme	Marks	AOs
9(a)	$\frac{1}{1-z}$	B1	2.2a
		(1)	
(b)(i)	$1+z+z^2+z^3+\ldots$ $=1+\left(\frac{1}{2}(\cos\theta+\mathrm{i}\sin\theta)\right)+\left(\frac{1}{2}(\cos\theta+\mathrm{i}\sin\theta)\right)^2+\left(\frac{1}{2}(\cos\theta+\mathrm{i}\sin\theta)\right)^3+\ldots$ $=1+\frac{1}{2}(\cos\theta+\mathrm{i}\sin\theta)+\frac{1}{4}(\cos 2\theta+\mathrm{i}\sin 2\theta)+\frac{1}{8}(\cos 3\theta+\mathrm{i}\sin 3\theta)+\ldots$	M1	3.1a
	$\frac{1}{1-z}=\frac{1}{1-\frac{1}{2}(\cos\theta+\mathrm{i}\sin\theta)}\times\frac{1-\frac{1}{2}\cos\theta+\frac{1}{2}\mathrm{i}\sin\theta}{1-\frac{1}{2}\cos\theta+\frac{1}{2}\mathrm{i}\sin\theta}$ or $\frac{1}{1-z}=\frac{2}{2-(\cos\theta+\mathrm{i}\sin\theta)}\times\frac{2-(\cos\theta-\mathrm{i}\sin\theta)}{2-(\cos\theta-\mathrm{i}\sin\theta)}$	M1	3.1a
	$\left\{\frac{1}{2}(\sin\theta)+\frac{1}{4}(\sin 2\theta)+\frac{1}{8}(\sin 3\theta)+\ldots\right\}=\frac{\frac{1}{2}\sin\theta}{\left(1-\frac{1}{2}\cos\theta\right)^2+\left(\frac{1}{2}\sin\theta\right)^2}$ or $\left\{\frac{1}{2}(\sin\theta)+\frac{1}{4}(\sin 2\theta)+\frac{1}{8}(\sin 3\theta)+\ldots\right\}=\frac{2\sin\theta}{(2-\cos\theta)^2+(\sin\theta)^2}$	M1	2.1
	$\left(1-\frac{1}{2}\cos\theta\right)^2+\left(\frac{1}{2}\sin\theta\right)^2=1-\cos\theta+\frac{1}{4}\cos^2\theta+\frac{1}{4}\sin^2\theta$ $=\frac{5}{4}-\cos\theta$ or $(2-\cos\theta)^2+(\sin\theta)^2=4-4\cos\theta+\cos^2\theta+\sin^2\theta$ $=5-4\cos\theta$	M1	1.1b
	$\frac{1}{2}\sin\theta+\frac{1}{4}\sin 2\theta+\frac{1}{8}\sin 3\theta+\ldots=\frac{\frac{1}{2}\sin\theta}{\frac{5}{4}-\cos\theta}=\frac{2\sin\theta}{5-4\cos\theta}$ *	A1*	1.1b
	Alternative $1+z+z^2+z^3+\ldots$ $=1+\left(\frac{1}{2}(\cos\theta+\mathrm{i}\sin\theta)\right)+\left(\frac{1}{2}(\cos\theta+\mathrm{i}\sin\theta)\right)^2+\left(\frac{1}{2}(\cos\theta+\mathrm{i}\sin\theta)\right)^3+\ldots$ $=1+\frac{1}{2}(\cos\theta+\mathrm{i}\sin\theta)+\frac{1}{4}(\cos 2\theta+\mathrm{i}\sin 2\theta)+\frac{1}{8}(\cos 3\theta+\mathrm{i}\sin 3\theta)+\ldots$	M1	3.1a

	$\dfrac{1}{1-z}=\dfrac{1}{1-\frac{1}{2}\mathrm{e}^{\mathrm{i}\theta}}\times\dfrac{1-\frac{1}{2}\mathrm{e}^{-\mathrm{i}\theta}}{1-\frac{1}{2}\mathrm{e}^{-\mathrm{i}\theta}}$	M1	3.1a
	$\dfrac{1-\frac{1}{2}\mathrm{e}^{-\mathrm{i}\theta}}{1-\frac{1}{4}\mathrm{e}^{\mathrm{i}\theta}-\frac{1}{4}\mathrm{e}^{-\mathrm{i}\theta}+\frac{1}{4}}=\dfrac{4-2\mathrm{e}^{-\mathrm{i}\theta}}{5-2\left(\mathrm{e}^{\mathrm{i}\theta}+\mathrm{e}^{-\mathrm{i}\theta}\right)}=\dfrac{4-2(\cos\theta-\mathrm{i}\sin\theta)}{5-2(2\cos\theta)}$	M1	2.1
	Select the imaginary part $\dfrac{2\sin\theta}{5-4\cos\theta}$	M1	1.1b
	$\dfrac{1}{2}\sin\theta+\dfrac{1}{4}\sin 2\theta+\dfrac{1}{8}\sin 3\theta+...=\dfrac{2\sin\theta}{5-4\cos\theta}$ *	A1*	1.1b
		(5)	
(b)(ii)	$\dfrac{1-\frac{1}{2}\cos\theta}{\frac{5}{4}-\cos\theta}=0\Rightarrow\cos\theta=2$	M1	3.1a
	As $(-1\le)\cos\theta\le 1$ therefore there is no solution to $\cos\theta=2$ so there will also be a real part, hence the sum cannot be purely imaginary.	A1	2.4
	Alternative 1 Imaginary part is $\dfrac{4-2\cos\theta}{5-4\cos\theta}=\dfrac{1}{2}+\dfrac{3}{2(5-4\cos\theta)}$	M1	3.1a
	$-1\le\cos\theta\le 1$ therefore $\dfrac{1}{6}<\dfrac{3}{2(5-4\cos\theta)}<\dfrac{3}{2}$ so sum must contain real part	A1	2.4
	Alternative 2 $\dfrac{1}{1-z}=k\mathrm{i}\Rightarrow z=1+\dfrac{\mathrm{i}}{k}$	M1	3.1a
	mod $z>1$ contradiction hence cannot be purely imaginary	A1	2.4
		(2)	
			(8 marks)

Notes:

(a)
B1: See scheme

(b)(i)

M1: Substitutes $z=\frac{1}{2}(\cos\theta+\mathrm{i}\sin\theta)$ into at least 3 terms of the series and applies de Moivre's theorem.

M1: Substitutes $z=\frac{1}{2}(\cos\theta+\mathrm{i}\sin\theta)$ into their answer to part (a) and rationalises the denominator.

M1: Equates the imaginary terms.

M1: Multiplies out the denominator and simplifies by using the identity $\cos^2\theta+\sin^2\theta=1$

A1*: cso**.** Achieves the printed answer having substituted $z=\frac{1}{2}(\cos\theta+\mathrm{i}\sin\theta)$ into 4 terms of the series. Alternative **M1:** Substitutes $z=\frac{1}{2}(\cos\theta+\mathrm{i}\sin\theta)$ into at least 3 terms of the series and applies de Moivre's theorem. **M1:** Substitutes $z=\frac{1}{2}\mathrm{e}^{\mathrm{i}\theta}$ into their answer to part (a) and rationalises the denominator. **M1:** Uses $\mathrm{e}^{-\mathrm{i}\theta}=\cos\theta-\mathrm{i}\sin\theta$ and $\mathrm{e}^{\mathrm{i}\theta}+\mathrm{e}^{-\mathrm{i}\theta}=2\cos\theta$ to express in terms of $\sin\theta$ and $\cos\theta$ **M1:** Select the imaginary terms. **A1*:** cso Achieves the printed answer having substituted $z=\frac{1}{2}(\cos\theta+\mathrm{i}\sin\theta)$ into 4 terms of the series.
(b)(ii) **M1:** Setting the real part of the series = 0 and rearranges to find $\cos\theta=...$ **A1:** See scheme
Alternative 1 **M1:** Rearranges imaginary part so that $\cos\theta$ only appears once **A1:** Uses $-1\le\cos\theta\le1$ to show that the sum must always be positive so must contain a real part
Alternative 2 **M1:** Sets sum as purely imaginary and rearranges to make z the subject **A1:** Shows a contradiction and draws an appropriate conclusion