

6. The curve C has equation

$$r = a(p + 2 \cos \theta) \quad 0 \leq \theta < 2\pi$$

where a and p are positive constants and $p > 2$

There are exactly four points on C where the tangent is perpendicular to the initial line.

(a) Show that the range of possible values for p is

$$2 < p < 4 \tag{5}$$

(b) Sketch the curve with equation

$$r = a(3 + 2 \cos \theta) \quad 0 \leq \theta < 2\pi \quad \text{where } a > 0 \tag{1}$$

John digs a hole in his garden in order to make a pond.

The pond has a uniform horizontal cross section that is modelled by the curve with equation

$$r = 20(3 + 2 \cos \theta) \quad 0 \leq \theta < 2\pi$$

where r is measured in centimetres.

The depth of the pond is 90 centimetres.

Water flows through a hosepipe into the pond at a rate of 50 litres per minute.

Given that the pond is initially empty,

(c) determine how long it will take to completely fill the pond with water using the hosepipe, according to the model. Give your answer to the nearest minute. (7)

(d) State a limitation of the model. (1)