Figure 1

Solutions based entirely on graphical or numerical methods are not acceptable.

 $y = \operatorname{arsinh} x$

(3)

(6)

Figure 1 shows a sketch of part of the curve with equation
$$y = \operatorname{arsinh} x$$
 $x \ge 0$

and the straight line with equation $y = \beta$

The line and the curve intersect at the point with coordinates (α, β)

Given that
$$\beta = \frac{1}{2} \ln 3$$

(a) show that
$$\alpha = \frac{1}{\sqrt{3}}$$

The finite region R, shown shaded in Figure 1, is bounded by the curve with equation $y = \operatorname{arsinh} x$, the y-axis and the line with equation $y = \beta$

The region R is rotated through 2π radians about the y-axis.

(b) Use calculus to find the exact value of the volume of the solid generated.