Scheme	Marks	AOs
$4\cosh^{3} x - 3\cosh x = 4\left(\frac{e^{x} + e^{-x}}{2}\right)^{3} - 3\left(\frac{e^{x} + e^{-x}}{2}\right)$	M1	1.2
$\equiv 4 \left(\frac{e^{3x} + 3e^x + 3e^{-x} + e^{-3x}}{8} \right) - 3 \left(\frac{e^x + e^{-x}}{2} \right)$	M1	1.1b
$= \frac{e^{3x}}{2} + \frac{3e^x}{2} + \frac{3e^{-x}}{2} + \frac{e^{-3x}}{2} - \frac{3e^x}{2} - \frac{3e^{-x}}{2} = \frac{e^{3x} + e^{-3x}}{2} = \cosh 3x^*$	A1*	2.1
	(3)	
$\cosh 3x = 9\cosh x \Rightarrow 4\cosh^3 x - 3\cosh x = 9\cosh x$ $\cosh^2 x = 3$	M1	3.1a
$\cosh x = \sqrt{3} \Rightarrow x = \ln\left(\sqrt{3} + \sqrt{\left(\sqrt{3}\right)^2 - 1}\right)$	M1	1.1b
$x = \ln\left(\sqrt{3} + \sqrt{2}\right) \text{ or } x = \ln\left(\sqrt{3} - \sqrt{2}\right)$	A1	1.1b
$x = \ln\left(\sqrt{3} + \sqrt{2}\right)$ and $x = \ln\left(\sqrt{3} - \sqrt{2}\right)$		
With no "solutions" being found by attempts to solve $\cosh x = 0$ or	A1	2.3
$\cosh x = -\sqrt{3}$		
	(4)	
(7 marks)		
Notes		
(a) M1: Recalls the definition of $\cosh x$ in terms of exponentials and substitutes M1: Expands the cubed bracket correctly A1*: Correct proof with no errors (b) M1: Uses the result from part (a) and collects terms to make progress in solving the equation M1: Recalls the definition of \cosh in terms of e or uses the definition of $\cosh^{-1}x$ A1: One correct solution A1: Both correct solutions and no others from $\cosh x = 0$ or $\cosh x = -\sqrt{3}$		
	$4\cosh^{3}x - 3\cosh x = 4\left(\frac{e^{x} + e^{-x}}{2}\right)^{3} - 3\left(\frac{e^{x} + e^{-x}}{2}\right)$ $= 4\left(\frac{e^{3x} + 3e^{x} + 3e^{-x} + e^{-3x}}{8}\right) - 3\left(\frac{e^{x} + e^{-x}}{2}\right)$ $= \frac{e^{3x}}{2} + \frac{3e^{x}}{2} + \frac{3e^{-x}}{2} + \frac{e^{-3x}}{2} - \frac{3e^{x}}{2} - \frac{3e^{-x}}{2} = \frac{e^{3x} + e^{-3x}}{2} = \cosh 3x^{*}$ $\cosh 3x = 9\cosh x \Rightarrow 4\cosh^{3}x - 3\cosh x = 9\cosh x$ $\cosh^{2}x = 3$ $\cosh x = \sqrt{3} \Rightarrow x = \ln\left(\sqrt{3} + \sqrt{\left(\sqrt{3}\right)^{2} - 1}\right)$ $x = \ln\left(\sqrt{3} + \sqrt{2}\right) \text{ or } x = \ln\left(\sqrt{3} - \sqrt{2}\right)$ $x = \ln\left(\sqrt{3} + \sqrt{2}\right) \text{ and } x = \ln\left(\sqrt{3} - \sqrt{2}\right)$ With no "solutions" being found by attempts to solve $\cosh x = 0$ or $\cosh x = -\sqrt{3}$ Notes Notes Notes the result from part (a) and collects terms to make progress in solving the result from part (a) and collects terms to make progress in solving the definition of cosh in terms of e or uses the definition of cosh-1x	$4\cosh^{3}x - 3\cosh x = 4\left(\frac{e^{x} + e^{-x}}{2}\right)^{3} - 3\left(\frac{e^{x} + e^{-x}}{2}\right) \qquad M1$ $= 4\left(\frac{e^{3x} + 3e^{x} + 3e^{-x} + e^{-3x}}{8}\right) - 3\left(\frac{e^{x} + e^{-x}}{2}\right) \qquad M1$ $= \frac{e^{3x}}{2} + \frac{3e^{x}}{2} + \frac{3e^{-x}}{2} + \frac{e^{-3x}}{2} - \frac{3e^{x}}{2} - \frac{3e^{-x}}{2} = \frac{e^{3x} + e^{-3x}}{2} = \cosh 3x^{*} \qquad A1^{*}$ $\cosh 3x = 9\cosh x \Rightarrow 4\cosh^{3}x - 3\cosh x = 9\cosh x$ $\cosh^{2}x = 3$ $\cosh x = \sqrt{3} \Rightarrow x = \ln\left(\sqrt{3} + \sqrt{\left(\sqrt{3}\right)^{2} - 1}\right) \qquad M1$ $x = \ln\left(\sqrt{3} + \sqrt{2}\right) \text{ or } x = \ln\left(\sqrt{3} - \sqrt{2}\right) \qquad A1$ $x = \ln\left(\sqrt{3} + \sqrt{2}\right) \text{ and } x = \ln\left(\sqrt{3} - \sqrt{2}\right)$ With no "solutions" being found by attempts to solve $\cosh x = 0$ or $\cosh x = -\sqrt{3}$ (4) Notes Cosh the definition of $\cosh x$ in terms of exponentials and substitutes and the cubed bracket correctly expect proof with no errors the result from part (a) and collects terms to make progress in solving the equation of the cosh in terms of e or uses the definition of $\cosh^{-1}x$ correct solution