Question	Scheme	Marks	AOs
7(a)	$4\cosh^3 x - 3\cosh x \equiv 4\left(\dfrac{e^x + e^{-x}}{2}\right)^3 - 3\left(\dfrac{e^x + e^{-x}}{2}\right)$	M1	1.2
	$\equiv 4\left(\dfrac{e^{3x} + 3e^x + 3e^{-x} + e^{-3x}}{8}\right) - 3\left(\dfrac{e^x + e^{-x}}{2}\right)$	M1	1.1b
	$= \dfrac{e^{3x}}{2} + \dfrac{3e^x}{2} + \dfrac{3e^{-x}}{2} + \dfrac{e^{-3x}}{2} - \dfrac{3e^x}{2} - \dfrac{3e^{-x}}{2} \equiv \dfrac{e^{3x} + e^{-3x}}{2} \equiv \cosh 3x$ *	A1*	2.1
		(3)	
(b)	$\cosh 3x = 9\cosh x \Rightarrow 4\cosh^3 x - 3\cosh x = 9\cosh x$ $\cosh^2 x = 3$	M1	3.1a
	$\cosh x = \sqrt{3} \Rightarrow x = \ln\left(\sqrt{3} + \sqrt{\left(\sqrt{3}\right)^2 - 1}\right)$	M1	1.1b
	$x = \ln\left(\sqrt{3} + \sqrt{2}\right)$ **or** $x = \ln\left(\sqrt{3} - \sqrt{2}\right)$	A1	1.1b
	$x = \ln\left(\sqrt{3} + \sqrt{2}\right)$ **and** $x = \ln\left(\sqrt{3} - \sqrt{2}\right)$ With no "solutions" being found by attempts to solve $\cosh x = 0$ or $\cosh x = -\sqrt{3}$	A1	2.3
		(4)	
			(7 marks)

Notes

(a)
M1: Recalls the definition of cosh$x$ in terms of exponentials and substitutes
M1: Expands the cubed bracket correctly
A1*: Correct proof with no errors
(b)
M1: Uses the result from part (a) and collects terms to make progress in solving the equation
M1: Recalls the definition of cosh in terms of e or uses the definition of $\cosh^{-1} x$
A1: One correct solution
A1: Both correct solutions and no others from $\cosh x = 0$ or $\cosh x = -\sqrt{3}$