

9. The watering of crops on a farm is thought to affect the concentration of nitrates in a nearby river. In a study, the concentration of nitrates in the river is measured at a point downstream from the farm.

The concentration of nitrates is modelled by the differential equation

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = e^{-t} + 1$$

where y is the concentration in milligrams per litre, t hours after the crops were watered.

- (a) Find a general solution for the concentration of nitrates after time t hours.

(6)

Initially

- the concentration of nitrates was measured as 1 milligram per litre,
- according to the model, the concentration was increasing at a rate of 9 milligram per litre every hour.

- (b) Find the particular solution for the concentration of nitrates after t hours.

(3)

- (c) Hence determine the maximum concentration of nitrates after the crops are watered.

(3)

The concentration of nitrates is believed to return to its initial concentration 8 hours after the crops are watered.

- (d) State, with justification, whether this is supported by the model.

(2)