Question	Scheme	Marks	AOs
3(a)	$n=1 \Rightarrow \mathbf{M}^1 = \begin{pmatrix} 3^1 & \frac{a}{2}(3^1-1) \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & a \\ 0 & 1 \end{pmatrix}$ {So the result is true for $n=1$}	B1	2.2a
	Assume true for $n=k$ Or assume $\mathbf{M}^n$ or $\begin{pmatrix} 3 & a \\ 0 & 1 \end{pmatrix}^k = \begin{pmatrix} 3^k & \frac{a}{2}(3^k-1) \\ 0 & 1 \end{pmatrix}$	M1	2.4
	A correct method to find an expression for $n=k+1$ $\begin{pmatrix} 3 & a \\ 0 & 1 \end{pmatrix}^{k+1} = \begin{pmatrix} 3^k & \frac{a}{2}(3^k-1) \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 3 & a \\ 0 & 1 \end{pmatrix}$ or $\begin{pmatrix} 3 & a \\ 0 & 1 \end{pmatrix}^{k+1} = \begin{pmatrix} 3 & a \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 3^k & \frac{a}{2}(3^k-1) \\ 0 & 1 \end{pmatrix}$	M1	1.1b
	$\begin{pmatrix} 3(3^k) & a(3^k)+\frac{a}{2}(3^k-1) \\ 0 & 1 \end{pmatrix}$ or $\begin{pmatrix} 3(3^k) & 3\times\frac{a}{2}(3^k-1)+a \\ 0 & 1 \end{pmatrix}$	A1	1.1b
	$\begin{pmatrix} 3^{k+1} & \frac{a}{2}[2(3^k)+(3^k-1)] \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 3^{k+1} & \frac{a}{2}[3(3^k)-1] \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 3^{k+1} & \frac{a}{2}[3^{k+1}-1] \\ 0 & 1 \end{pmatrix}$ $\begin{pmatrix} 3(3^k) & 3\times\frac{a}{2}(3^k-1)+a \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 3^{k+1} & \frac{a}{2}(3(3^k-1)+2) \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 3^{k+1} & \frac{a}{2}(3^{k+1}-1) \\ 0 & 1 \end{pmatrix}$	A1	2.1
	If true for $n=k$ **then** true for $n=k+1$ and as it is **true** for $n=1$ the statement is **true for** all (positive integers) ***n***	A1	2.4
		(6)	
(b)(i)	$det(\mathbf{M}^n)=3^n$ or $det(\mathbf{M})=3$	B1	1.1b
	Uses $5\times det(\mathbf{M}^n)=1215 \Rightarrow p^n=q \Rightarrow n=\ldots$ $5\times 3^n=1215 \Rightarrow 3^n=243 \Rightarrow n=\ldots$	M1	3.1a
	$n=5$	A1	1.1b
(ii)	$\begin{pmatrix} 3^n & \frac{a}{2}(3^n-1) \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 123 \\ -2 \end{pmatrix} \Rightarrow 2(3^n)-2\frac{a}{2}(3^n-1)=123$ $\Rightarrow a=\ldots$	M1	1.1b

	$\begin{pmatrix} 243 & \frac{a}{2}(243-1) \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 123 \\ -2 \end{pmatrix} \Rightarrow 2(243) - 2\frac{a}{2}(243-1) = 123 \Rightarrow a = \ldots$ $\frac{1}{243}\begin{pmatrix} 1 & -\frac{a}{2}(243-1) \\ 0 & a \end{pmatrix}\begin{pmatrix} 123 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \end{pmatrix} \Rightarrow \frac{123 - 2\frac{a}{2}(243-1)}{243} = -2 \Rightarrow a = \ldots$		
	$a = 1.5$	A1	1.1b
		(5)	
			(11 marks)

Notes:

(a)

B1: Shows that the result holds for $n = 1$. Must see substitution in the RHS minimum required is $\begin{pmatrix} 3 & \frac{a}{2}(3-1) \\ 0 & 1 \end{pmatrix}$ and reaches $\begin{pmatrix} 3 & a \\ 0 & 1 \end{pmatrix}$

M1: Assumes the result is true for some value of $n = k$. Assume (true for) $n = k$ is sufficient.

Alternatively states assume $\mathbf{M}^n$ or $\begin{pmatrix} 3 & a \\ 0 & 1 \end{pmatrix}^k = \begin{pmatrix} 3^k & \frac{a}{2}(3^k - 1) \\ 0 & 1 \end{pmatrix}$

M1: Sets up a matrix multiplication of their assumed result multiplied by the original matrix, either way round. Allow a slip as long as the intention is clear.

A1: Achieves a correct un-simplified matrix

A1: Reaches a correct simplified matrix with **no errors, the correct un-simplified matrix seen previously and at least one intermediate line which must be correct.**

A1: Correct conclusion. This mark is dependent on all previous marks except B mark but $n = 1$ must have been attempted. It is gained by conveying the ideas of **all four bold** points either at the end of their solution or as a narrative in their solution. Condone $n \in \mathbb{Z}$

(b)(i)

B1: States correct determinant. This can be implied by a correct equation

M1: Correct method to find a value of n using $5 \times$ 'their $det(\mathbf{M}^n)$' $= 1215$ which involves solving an index equation of the form $p^n = q$ where $n > 1$

A1: $n = 5$

(ii)

M1: Sets up an equation by multiplying the matrix $\mathbf{M}^n$ by $\begin{pmatrix} 2 \\ -2 \end{pmatrix}$ setting equal to $\begin{pmatrix} 123 \\ -2 \end{pmatrix}$ and reaches a value for a. You may just see $2(3^n) - 2\frac{a}{2}(3^n - 1) = 123 \Rightarrow a = \ldots$

Follow through on their value for n.

A1: $a = 1.5$