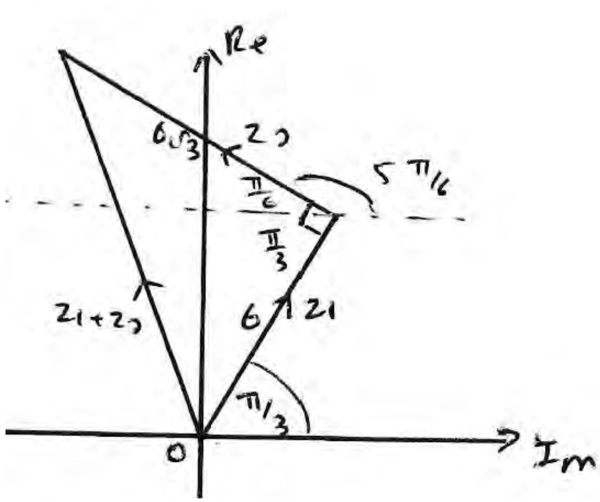
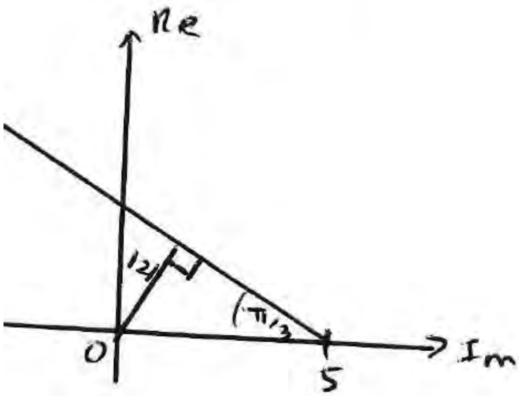
Question	Scheme		Marks	AOs
4(i)	$z_1 = 6\left[\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right] = \ldots\{3+3\sqrt{3}i\}$ $z_2 = 6\sqrt{3}\left[\cos\left(\frac{5\pi}{6}\right) + i\sin\left(\frac{5\pi}{6}\right)\right] = \ldots\{-9+3\sqrt{3}i\}$ $\{z_1+z_2=\}(3+3\sqrt{3}i)+(-9+3\sqrt{3}i)=\ldots\{-6+6\sqrt{3}i\}$ Or $\{z_1+z_2=\}6\left[\cos\left(\frac{\pi}{3}\right)+i\sin\left(\frac{\pi}{3}\right)\right]+6\sqrt{3}\left[\cos\left(\frac{5\pi}{6}\right)+i\sin\left(\frac{5\pi}{6}\right)\right] = a+bi$ where a and b are constants, the trig function must be evaluated		M1	3.1a
	Clearly show the method to find modulus **and** argument for z_1+z_2 $arg(z_1+z_2) = \pi - \tan^{-1}\left(\frac{6\sqrt{3}}{6}\right)$ or $\tan^{-1}\left(\frac{6\sqrt{3}}{-6}\right) = \ldots\left\{\frac{2\pi}{3}\right\}$ **and** $\lvert z_1+z_2\rvert = \sqrt{6^2+(6\sqrt{3})^2} = \ldots\{12\}$	**Alternative 1** $-6+6\sqrt{3}i = 12\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)$ $= 12\left(\cos\left(\frac{2\pi}{3}\right)+i\sin\left(\frac{2\pi}{3}\right)\right)$ **Alternative 2** $12e^{\frac{2\pi}{3}i} = 12\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)$ $=\ldots\{-6+6\sqrt{3}i\}$	dM1	2.1
	$z_1+z_2 = 12e^{\frac{2\pi}{3}i}$ *	$12e^{\frac{2\pi}{3}i} = -6+6\sqrt{3}i$ Therefore $z_1+z_2 = 12e^{\frac{2\pi}{3}i}$*	A1*	1.1b
			(3)	
	Alternative 3 $z_1+z_2 = 6e^{\frac{\pi}{3}i}+6\sqrt{3}e^{\frac{5\pi}{6}i}$ $= 12\left[\frac{1}{2}\cos\left(\frac{\pi}{3}\right)+\frac{1}{2}i\sin\left(\frac{\pi}{3}\right)+\frac{\sqrt{3}}{2}\cos\left(\frac{5\pi}{6}\right)+\frac{\sqrt{3}}{2}i\sin\left(\frac{5\pi}{6}\right)\right]$		M1	3.1a
	$12\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right) = 12\left(\cos\left(\frac{2\pi}{3}\right)+i\sin\left(\frac{2\pi}{3}\right)\right)$		dM1	2.1
	$z_1+z_2 = 12e^{\frac{2\pi}{3}i}$*		A1*	1.1b
			(3)	
	Alternative 4 $z_1+z_2 = 6e^{\frac{\pi}{3}i}+6\sqrt{3}e^{\frac{5\pi}{6}i} = 6e^{\frac{\pi}{3}i}\left(1+\sqrt{3}e^{\frac{\pi}{2}i}\right) = 6e^{\frac{\pi}{3}i}(1+\sqrt{3}i)$		M1	
	Either $r = \sqrt{1^2+\left(\sqrt{3}\right)^2} = 2$ **and** $arg = \arctan\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$		dM1	

	Or $6e^{\frac{\pi}{3}i}\left(1+\sqrt{3}i\right) = 12e^{\frac{\pi}{3}i}\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i()e^{\frac{\pi}{3}i}\left(cos\left(\frac{\pi}{3}\right)+i\,sin\left(\frac{\pi}{3}\right)\right)\right)$		
	$z_1+z_2 = 12e^{\frac{\pi}{3}i}e^{\frac{\pi}{3}i} = 12e^{\frac{2\pi}{3}i}*$	A1*	
		(3)	
	Alternative 5 Uses geometry to show that $z_1,\ z_2$ and z_1+z_2 form a right-angled triangle 	M1	3.1a
	$arg(z_1+z_2) = \frac{\pi}{3}+tan^{-1}\left(\frac{6\sqrt{3}}{6}\right) =\ldots\left\{\frac{2\pi}{3}\right\}$ $\lvert z_1+z_2\rvert = \sqrt{(6)^2+(6\sqrt{3})^2} =\ldots\{12\}$	dM1	1.1b
	$z_1+z_2 = 12e^{\frac{2\pi}{3}i}\ *$	A1*	1.1b
		(3)	
(ii)		M1	3.1a
	$sin\left(\frac{\pi}{3}\right) = \frac{\lvert z\rvert}{5} \Rightarrow \lvert z\rvert =\ldots$	M1	1.1b
	$\lvert z\rvert = \frac{5\sqrt{3}}{2}$	A1	1.1b
		(3)	

	Alternative 1 Gradient = $-\tan\left(\frac{\pi}{3}\right)$ $c = 5\tan\left(\frac{\pi}{3}\right)$ leading to $y = -\sqrt{3}x + 5\sqrt{3}$ or $\tan\left(\frac{\pi}{3}\right) = \frac{y}{5-x}$ $\lvert z\rvert^2 = x^2 + y^2 = x^2 + (-\sqrt{3}x + 5\sqrt{3})^2 = 4x^2 - 30x + 75$ $\frac{d\lvert z\rvert^2}{dx} = 8x - 30 = 0 \Rightarrow x = \ldots\{3.75\}$ or $\lvert z\rvert^2 = 4(x - 3.75)^2 + 18.75 \Rightarrow x = \ldots\ldots\{3.75\}$	M1	3.1a
	$\lvert z\rvert = \sqrt{4(\text{their}3.75)^2 - 30(\text{their}3.75) + 75}$	M1	1.1b
	$\lvert z\rvert = \frac{5\sqrt{3}}{2}$	A1	1.1b
		(3)	
	Alternative 2 Gradient = $-\tan\left(\frac{\pi}{3}\right)$ $c = 5\tan\left(\frac{\pi}{3}\right)$ leading to $y = -\sqrt{3}x + 5\sqrt{3}$ Perpendicular line through the origin $y = \frac{1}{\sqrt{3}}x$ and find the point of intersection of the two lines $\left(\frac{15}{4}, \frac{5\sqrt{3}}{4}\right)$	M1	3.1a
	Finds the distance from the origin to their point of intersection $\lvert z\rvert = \sqrt{\left(\text{their } \frac{15}{4}\right)^2 + \left(\text{their } \frac{5\sqrt{3}}{4}\right)^2} = \ldots$	M1	1.1b
	$\lvert z\rvert = \frac{5\sqrt{3}}{2}$	A1	1.1b
		(3)	
(6 marks)			

Notes:

(i)
M1: A complete method to find both $z_1$and z_2 in the form $a + bi$and adds them together.
dM1: Dependent on previous method mark, finds the modulus and argument of $z_1 + z_2$. They must show their method, just stating modulus = 12 and argument = $\frac{2\pi}{3}$ is not sufficient as this is a show question.
Alternative 1: Factorises out 12 and find the argument

Alternative 2: uses $12e^{\frac{2\pi}{3}i} = 12\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) = \ldots$
A1*: Achieves the correct answer following no errors or omissions**.**
Alternatively shows that $12e^{\frac{2\pi}{3}i} = -6 + 6\sqrt{3}i$and concludes therefore $z_1 + z_2 = 12e^{\frac{2\pi}{3}i}$*

<u>Alternative 3</u>

M1: Factorises out 12 and writes in the form $12\left[\ldots \boldsymbol{cos}\left(\frac{\pi}{3}\right)+\ldots \boldsymbol{i}\,\boldsymbol{sin}\left(\frac{\pi}{3}\right)+\ldots \boldsymbol{cos}\left(\frac{5\pi}{6}\right)+\ldots \boldsymbol{i}\,\boldsymbol{sin}\left(\frac{5\pi}{6}\right)\right]$

dM1: Dependent on previous mark**.** Writes in the form $12(a+bi)$ leading to the form $12(cos\,\theta + i\,sin\,\theta)$

A1*: Achieves the correct answer following no errors or omissions**.**

<u>**Alternative 4**</u>

M1: Factorises out 6 and writes in the form $6e^{\frac{\pi}{3}i}\left(1+\sqrt{3}e^{\frac{\pi}{2}i}\right) = 6e^{\frac{\pi}{3}i}(1+ai)$

dM1: Dependent on previous method mark, finds the modulus and argument of $(1+ai)$ or $12(a+bi)$ leading to the form $12(cos\,\theta + i\,sin\,\theta)$

A1*: Achieves the correct answer following no errors or omissions**.**

<u>**Alternative 5**</u>

M1: Draws a diagram to show that z_1, z_2 and $z_1 + z_2$ form a right-angled triangle.

dM1: Dependent on previous method mark, finds the modulus and argument of $z_1 + z_2$

A1*: Achieves the correct answer following no errors or omissions**.**

Note: Writing $arg(z_1+z_2) = arctan\left(\frac{6\sqrt{3}}{-6}\right) = -\frac{\pi}{3}$ therefore $arg(z_1+z_2) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ with no diagram or finding $z_1 + z_2$ is **M0dM0A0**

(ii)

M1: Draws a diagram and recognises that the shortest distance will form a right-angled triangle.

M1: Uses trigonometry to find the shortest length.

A1: Correct exact value.

<u>**Alternative 1**</u>

M1: Finds the equation of the half-line by attempting $m = -\,tan\left(\frac{\pi}{3}\right)$ $c = 5\,tan\left(\frac{\pi}{3}\right)$. Finds $x^2 + y^2$ in terms of *x*, differentiates, sets = 0 and finds the value of *x*.

M1: Uses their value of *x* to find the minimum value of $\sqrt{x^2+y^2}$

A1: Correct exact value.

<u>**Alternative 2**</u>

M1: Finds the equation of the half-line by attempting $m = -\,tan\left(\frac{\pi}{3}\right)$ $c = 5\,tan\left(\frac{\pi}{3}\right)$. Finds the equation of the line perpendicular which passes through the origin. Finds the point of intersection of the lines

M1: Finds the distance from the origin to their point of intersection

A1: Correct exact value.