6(a)

$$
4 x^{3}+p x^{2}-14 x+q=0 \Rightarrow x^{3}+\frac{p}{4} x^{2}-\frac{14}{4} x+\frac{q}{4}=0
$$

$$
\alpha+\beta+\gamma=-\frac{p}{4} \alpha \beta+\alpha \gamma+\beta \gamma=-\frac{14}{4} \text { or }-\frac{7}{2}
$$

$$
(\alpha+\beta+\gamma)^{2}=\alpha^{2}+\beta^{2}+\gamma^{2}+2(\alpha \beta+\alpha \gamma+\beta \gamma)
$$

$$
\left(-\frac{p}{4}\right)^{2}=16+2\left(-\frac{7}{2}\right) \Rightarrow p=\ldots
$$

or

$$
(\alpha+\beta+\gamma)^{2}-2(\alpha \beta+\alpha \gamma+\beta \gamma)=\alpha^{2}+\beta^{2}+\gamma^{2}
$$

$$
\left(-\frac{p}{4}\right)^{2}-2\left(-\frac{7}{2}\right)=16 \Rightarrow p=\ldots
$$

(b) $\quad \frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=\frac{\beta \gamma+\alpha \gamma+\alpha \beta}{\alpha \beta \gamma}$

$$
\frac{\left(-\frac{7}{2}\right)}{\left(\frac{-q}{4}\right)}=\frac{14}{3} \Rightarrow q=\ldots
$$

$$
\begin{equation*}
q=3 \tag{3}
\end{equation*}
$$

## Alternative

| $4\left(\frac{1}{w}\right)^{3}+12\left(\frac{1}{w}\right)^{2}-14\left(\frac{1}{w}\right)+q\{=0\}$ | M1 | 1.1b |
| :---: | :---: | :---: |
| $q w^{3}-14 w^{2}+12 w+4=0 \Rightarrow \frac{14}{3}=-\frac{-14}{q} \Rightarrow q=\ldots$ | M1 | 1.1b |
| $q=3$ | A1 | 1.1b |
|  | (3) |  |
| $\begin{aligned} & (\alpha-1)(\beta-1)(\gamma-1)=\ldots \\ & =\alpha \beta \gamma-(\alpha \beta+\alpha \gamma+\beta \gamma)+(\alpha+\beta+\gamma)-1 \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ | $\begin{aligned} & 1.1 \mathrm{a} \\ & 1.1 \mathrm{~b} \end{aligned}$ |
| $=\left(-\frac{\text { their } 3}{4}\right)-\left(-\frac{7}{2}\right)+\left(-\frac{12}{4}\right)-1=\ldots$ | dM1 | 1.1b |
| $=-\frac{5}{4}$ | A1 | 1.1b |
|  | (4) |  |
| ( +1$)^{3}+12(x+1)^{2}-14(x+1)+$ '3' $\{=0\}$ or substitutes in 1 | M1 | 1.1a |
| $4+\ldots 12+\ldots-14+{ }^{\prime} 3^{\prime}=5 \text { or } 4 x^{3}+24 x^{2}+22 x+2+$ | A1ft | 1.1b |


|  | $=-\frac{\text { 'their constant' }_{4}^{4}}{}$ | dM 1 | 1.1 b |
| :---: | :---: | :---: | :---: |
|  | $=-\frac{5}{4}$ | A 1 | 1.1 b |
|  |  |  |  |

(10 marks)

## Notes:

(a)

B1: Identifies the correct values for the sum and pair sum. This may be implied by substituting into an equation, it must be clear
M1: Uses the correct identity and values of their sum and their pair sum to find a value of $p$
A1*: $p=12$ cso there is no need to see a reason
(b)

M1: Establishes a correct identity
M1: Uses their identity and their pair sum and their product of roots to find a value of $q$. Condone a slip but the intention must be clear.
A1: $q=3$ Allow this mark from incorrect sign of both pair sum and product

## Alternative

M1: Uses $x=\frac{1}{w}$ the substitution
M1: Simplifies to an quartic equation of the form $a w^{3}+b w^{2}+c w+d=0$ and uses $\frac{14}{3}=-\frac{b}{a}$ to find a value for $q$
A1: $q=3$
(c)

M1: Attempts to multiply out the three brackets.
A1: Correct expansion.
dM1: Dependent on previous method. Substitutes in the value of their sum, pair sum and the value of their product as appropriate. Condone a slip but the intention must be clear
A1: Correct value

## Alternative

M1: Substitutes $(x+1)$ or $x=1$ into the cubic with their value of $q$. Allow the use of different letters e.g. $(w+1)$

A1ft: Correct constant terms, follow through on their value of $q$
dM1: Applies - 'their constant'
A1: Correct value

