Question	Scheme	Marks	AOs
7(a)	$x = r\cos\theta = (1+\tan\theta)\cos\theta = \cos\theta + \sin\theta$ $= \cos\theta + \tan\theta\cos\theta$ $\frac{dx}{d\theta} = \alpha(1+\tan\theta)\sin\theta + \beta\sec^2\theta\cos\theta$ or $\frac{dx}{d\theta} = \alpha\sin\theta + \beta\cos\theta$ $\frac{dx}{d\theta} = \alpha\sin\theta + \beta\sec^2\theta\cos\theta + \delta\tan\theta\sin\theta$	M1	3.1a
	$\frac{dx}{d\theta} = -(1+\tan\theta)\sin\theta + \sec^2\theta\cos\theta$ or $\frac{dx}{d\theta} = -\sin\theta + \cos\theta$ $\frac{dx}{d\theta} = -\sin\theta + \sec^2\theta\cos\theta - \tan\theta\sin\theta$ or $\frac{dx}{d\theta} = -\sin\theta + \sec\theta - \tan\theta\sin\theta$	A1	1.1b
	For example $\left\{\frac{dx}{d\theta} =\right\} -\sin\theta + \cos\theta = 0 \Rightarrow \tan\theta = 1 \Rightarrow \theta = \ldots$ $\left\{\frac{dx}{d\theta} =\right\} -\sin\theta + \cos\theta = 0 \Rightarrow \sin\theta = \cos\theta \Rightarrow \theta = \ldots$ $\left\{\frac{dx}{d\theta} =\right\} -\sin\theta + \cos\theta = \sqrt{2}\cos\left(\theta + \frac{\pi}{4}\right) = \theta = \ldots$ or $\left\{\frac{dx}{d\theta} =\right\} -(1+\tan\theta)\sin\theta + \sec^2\theta\cos\theta = 0$ $\Rightarrow -\sin\theta - \frac{\sin^2\theta}{\cos\theta} + \frac{1}{\cos\theta} = 0 \Rightarrow -\sin\theta + \frac{1-\sin^2\theta}{\cos\theta} = 0$ $\Rightarrow -\sin\theta + \cos\theta = 0 \Rightarrow \tan\theta = 1 \Rightarrow \theta = \ldots$ or $\left\{\frac{dx}{d\theta} =\right\} -\sin\theta - \tan\theta\sin\theta + \sec\theta = 0$ $\Rightarrow -\frac{1}{2}\sin 2\theta - \sin^2\theta + 1 = 0 \Rightarrow \sin 2\theta + 2\sin^2\theta - 1 = 1$ $\Rightarrow \sin 2\theta - \cos 2\theta = 1 \Rightarrow \sqrt{2}\sin\left(2\theta - \frac{\pi}{4}\right) = 1 \Rightarrow \theta = \ldots$ or $\left\{\frac{dx}{d\theta} =\right\} -\sin\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{4}\right) = 0$ $\left\{\frac{dx}{d\theta} =\right\} -\left(1+\tan\left(\frac{\pi}{4}\right)\right)\sin\left(\frac{\pi}{4}\right) + \sec^2\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right) = 0$ $\left\{\frac{dx}{d\theta} =\right\} -\sin\left(\frac{\pi}{4}\right) + \sec^2\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right) - \tan\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}\right) = 0$	dM1	3.1a
	$r = 1 + \tan\left(\frac{\pi}{4}\right) = 2$ therefore $A\left(2, \frac{\pi}{4}\right)$*	A1*	2.1
		(4)	
	Area bounded by the curve $= \frac{1}{2}\int(1+\tan\theta)^2 \ \{d\theta\}$	M1	3.1a

(b)	$= \frac{1}{2}\int (1 + 2\tan\theta + \tan^2\theta)\ \{d\theta\}$ $= \frac{1}{2}\int (1 + 2\tan\theta + [\sec^2\theta - 1])\ \{d\theta\} = \ldots$		
	$= \frac{1}{2}[2\ln\lvert\sec\theta\rvert + \tan\theta]$ or $\ln\lvert\sec\theta\rvert + \frac{1}{2}\tan\theta$ or $-\ln\cos\theta + \frac{1}{2}\tan\theta$ or $= \frac{1}{2}[-2\ln\lvert\cos\theta\rvert + \tan\theta]$	A1	1.1b
	$= \frac{1}{2}\left[2\ln\left\lvert\sec\left(\frac{\pi}{4}\right)\right\rvert + \tan\left(\frac{\pi}{4}\right)\right] - \frac{1}{2}[2\ln\lvert\sec(0)\rvert + \tan(0)]$ $= \left(\ln\left\lvert\sec\left(\frac{\pi}{4}\right)\right\rvert + \frac{1}{2}\tan\left(\frac{\pi}{4}\right)\right) - \left(\ln\lvert\sec 0\rvert + \frac{1}{2}\tan 0\right)$ $\left\{= \ln\sqrt{2} + \frac{1}{2}\right\}$	dM1	1.1b
	Area of triangle $= \frac{1}{2}xy = \frac{1}{2}\left(2\cos\frac{\pi}{4}\right)\left(2\sin\frac{\pi}{4}\right) = \ldots\left\{\frac{1}{2}\times\sqrt{2}\times\sqrt{2} = 1\right\}$ The equation of the tangent is $r = \sqrt{2}\sec\theta$ then applies Area bounded of triangle $= \frac{1}{2}\int_0^{\frac{\pi}{4}}(\sqrt{2}\sec\theta)^2\ \{d\theta\}$	M1	1.1b
	Finds the required area = area of triangle – area bounded by the curve $= 1 - \left[\ln\sqrt{2} + \frac{1}{2}\right]$ May be seen within an integral $= \frac{1}{2}\int(\sqrt{2}\sec\theta)^2\ \{d\theta\} - \frac{1}{2}\int(1 + \tan\theta)^2\ \{d\theta\}$	M1	3.1a
	$= \frac{1}{2}(1 - \ln 2)$ * cso	A1*	2.1
		(6)	
	Alternative Area bounded by the curve $= \frac{1}{2}\int(1 + \tan\theta)^2\ \{d\theta\}$ $= \frac{1}{2}\int(1 + 2\tan\theta + \tan^2\theta)\ \{d\theta\}$ let $u = \tan\theta \Rightarrow \frac{du}{d\theta} = \sec^2\theta$ Leading to $= \frac{1}{2}\int \frac{(1+2u+u^2)}{1+u^2}\{du\} = \frac{1}{2}\int 1 + \frac{2u}{1+u^2}\{du\} = \ldots$	M1	3.1a
	$\frac{1}{2}[u + \ln(1 + u^2)]$	A1	1.1b
	$\frac{1}{2}[(1 + \ln(1 + (1)^2)) - (0 + \ln 1)]$ or $\frac{1}{2}\left[\left(\tan\left(\frac{\pi}{4}\right) + \ln\left(1 + \tan^2\left(\frac{\pi}{4}\right)\right)\right) - (\tan(0) + \ln(1 + \tan^2(0)))\right]$ $\left\{= \frac{1}{2}\ln 2 + \frac{1}{2}\right\}$	dM1	1.1b
	Area of triangle $= \frac{1}{2}xy = \frac{1}{2}\left(2\cos\frac{\pi}{4}\right)\left(2\sin\frac{\pi}{4}\right) = \ldots\left\{\frac{1}{2}\times\sqrt{2}\times\sqrt{2} = 1\right\}$	M1	1.1b

	Finds the required area = area of triangle – area bounded by the curve $= 1 - \left[ln\sqrt{2} + \frac{1}{2}\right]$	M1	3.1a
	$= \frac{1}{2}(1 - ln\,2)$ *	A1*	2.1
		(6)	
			(10 marks)

Notes:

(a)

M1: Substitutes the equation of C into $x = r\cos\theta$ and differentiates to the required form

A1: Fully correct differentiation

dM1: Dependent on previous method mark. Sets their $\frac{dx}{d\theta} = 0$ and uses correct trig identities to find a value for θ. Alternatively substitutes $\theta = \frac{\pi}{4}$ into their $\frac{dx}{d\theta}$ and shows equals 0.

A1*: Shows that $r = 2$ and hence the polar coordinates $\left(2, \frac{\pi}{4}\right)$ from correct working

(b)

M1: Applies area $= \frac{1}{2}\int r^2\,\theta\;d\theta$, multiplies out, uses the identity $\pm 1 \pm tan^2\,\theta = sec^2\,\theta$ to get into an integrable form **and** integrates. Condone missing $d\theta$, limits are not required for this mark

A1: Correct integration. Note may include θ – θ if the one's were not cancelled earlier.

dM1: Dependent on the first method mark. Applies the limits of $\theta = 0$ and $\theta = \frac{\pi}{4}$ and subtracts the correct way round. Since substitution of the limit $\theta = 0$ is 0 so may be implied

M1: Correct method to find the area of triangle seen. This may be minimal but area = 1 only is M0, they need to show some method.

M1: Finds the required area = area of triangle – area bounded by the curve

A1*: Correct answer, with no errors or omissions. cso

Alternative

M1: Applies area $= \frac{1}{2}\int r^2\,\theta\;d\theta$, multiplies out, uses the substitution $u = tan\,\theta$ to get into an integrable form **and** integrates. Limits are not required for this mark

A1: Correct integration

dM1: Dependent on the first method mark. Applies the limits of $u = 0$ and $u = 1$ or substitutes back using $u = tan\,\theta$ and uses the limits $\theta = 0$ and $\theta = \frac{\pi}{4}$ and subtracts the correct way round. Since substitution of the limit $\theta = 0$ is 0 so may be implied

M1: Correct method to find the area of triangle

M1: Finds the required area = area of triangle – area bounded by the curve

A1*: Correct answer, with no errors or omissions. cso