Question	Scheme	Marks	AOs
8(a)	A complete method to use the scalar product of the direction vectors and the angle 120° to form an equation in a $$\frac{\begin{pmatrix}2\\a\\0\end{pmatrix}\bullet\begin{pmatrix}0\\1\\-1\end{pmatrix}}{\sqrt{2^2+a^2}\sqrt{1^2+(-1)^2}}=\cos 1\,20$$	M1	3.1b
	$$\frac{a}{\sqrt{4+a^2}\sqrt{2}}=-\frac{1}{2}$$	A1	1.1b
	$$2a=-\sqrt{4+a^2}\sqrt{2}\Rightarrow 4a^2=8+2a^2\Rightarrow a^2=4\Rightarrow a=\ldots$$	M1	1.1b
	$$a=-2$$	A1	2.2a
		(4)	
(b)	Any two of **i:** $-1+2\lambda=4$ (1) **j:** $5+\text{'their}-2\text{'}\lambda=-1+\mu$ (2) **k:** $2=3-\mu$ (3)	M1	3.4
	Solves the equations to find a value of $\lambda\left\{=\frac{5}{2}\right\}$ **and** $\mu\{=1\}$	M1	1.1b
	$r_1=\begin{pmatrix}-1\\5\\2\end{pmatrix}+\frac{5}{2}\begin{pmatrix}2\\\text{'their}-2\text{'}\\0\end{pmatrix}$ or $r_2=\begin{pmatrix}4\\-1\\3\end{pmatrix}+1\begin{pmatrix}0\\1\\-1\end{pmatrix}$	dM1	1.1b
	(4,0,2) or $\begin{pmatrix}4\\0\\2\end{pmatrix}$	A1	1.1b
	Checks the third equation e.g. $\lambda=\frac{5}{2}$: **L HS** $=5-2\lambda=5-5=0$ $\mu=1$: **R HS** $=-1+\mu=-1+1=0$ therefore **common point/intersect/consistent/tick** or substitutes the values of λ and μ into the relevant lines and achieves the same coordinate	B1	2.1
		(5)	
(c)	Full attempt to find the minimum distance from the point of intersection (nest) to the plane (ground) E.g. Minimum distance $=\frac{\lvert 2\times\text{'4'}+(-3)\times\text{'0'}+1\times\text{'2'}-2\rvert}{\sqrt{2^2+(-3)^2+1)^2}}=\ldots$ **Alternatively** $\mathbf{r}=\begin{pmatrix}\text{'4'}\\\text{'0'}\\\text{'2'}\end{pmatrix}+\lambda\begin{pmatrix}2\\-3\\1\end{pmatrix}$ $2(\text{'4'}+2\lambda)-3(\text{'0'}-3\lambda)+(\text{'2'}+\lambda)=2\Rightarrow$ $\lambda=\ldots\left\{-\frac{4}{7}\right\}$	M1 A1ft	3.1b 3.4

	$\mathbf{r} = \begin{pmatrix} '4' \\ '0' \\ '2' \end{pmatrix} + '-\frac{4}{7}'\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{20}{7} \\ \frac{12}{7} \\ \frac{10}{7} \end{pmatrix}$ Minimum distance $= \sqrt{\left(2 \times -\frac{4}{7}'\right)^2 + \left(-3 \times -\frac{4}{7}'\right)^2 + \left(1 \times -\frac{4}{7}\right)^2} = \ldots$ $= \sqrt{\left('4' - '\frac{20}{7}'\right)^2 + \left('0' - '\frac{12}{7}'\right)^2 + \left('2' - '\frac{10}{7}'\right)^2} = \ldots$		
	$\frac{8}{\sqrt{14}}$ or $\frac{4\sqrt{14}}{7}$ or awrt 2.1	A1	2.2b
		(3)	
	Alternative Find perpendicular distance from plane to the origin $2x - 3y + z = 2$ $\lvert n \rvert = \sqrt{2^2 + (-3)^2 + 1^2} = \sqrt{14}$ shortest distance $= \frac{2}{\sqrt{14}}$ Find perpendicular distance from the plane containing the point of intersection to the origin $2x - 3y + z = \begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = 10$ shortest distance $= \frac{10}{\sqrt{14}}$ Minimum distance $= \frac{10}{\sqrt{14}} - \frac{2}{\sqrt{14}}$	M1 A1ft	3.1b 3.4
	$\frac{8}{\sqrt{14}}$ or $\frac{4\sqrt{14}}{7}$ or awrt 2.1	A1	2.2b
		(3)	
(d)	For example Not reliable as the birds will not fly in a straight line Not reliable as angle between flights paths will not always be 120° Not reliable/reliable as the ground will not be flat/smooth Not reliable as bird's nest is not a point	B1	3.2b
		(1)	
			(13 marks)

Notes:

(a)

M1: See scheme, allow a sign slip and cos 60

A1: Correct simplified equation in a, cos 120 must be evaluated to – ½ and dot product calculated

Note: If the candidate states either $\left|\frac{a\cdot b}{|a||b|}\right| = \cos\theta$ or $\left|\frac{a}{\sqrt{4+a^2}\sqrt{2}}\right| = \cos 60$ then has the equation $\frac{a}{\sqrt{4+a^2}\sqrt{2}} = \frac{1}{2}$ award this mark. If the module of the dot product is not seen then award A0 for this equation.

dM1: Solve a quadratic equation for a, by squaring and solving an equation of the form $a^2 = K$ where $K > 0$ **A1:** Deduces the correct value of a from a correct equation. Must be seen in part (a) using the angle between the lines. **Alternative cross product method** **M1:** $\begin{vmatrix} 2 & a & 0 \\ 0 & 1 & -1 \end{vmatrix} = \sqrt{2^2+a^2}\sqrt{1^2+(-1)^2}\, sin\, 1\,20$ **A1:** $\sqrt{a^2+8} = \sqrt{4+a^2}\sqrt{2}\frac{\sqrt{3}}{2}$ Then as above **Note If they use the point of intersection to find a value for *a* this scores no marks**
(b) **M1:** Uses the model to write down any two correct equations **M1:** Solve two equations simultaneously to find a value for μ and λ **dM1**: Dependent on previous method mark. Substitutes μ and λ into a relevant equation. If no method shown two correct ordinates implies this mark. **A1:** Correct coordinates. May be seen in part (c) **B1**: Shows that the values of μ and λ give the same third coordinate or point of intersection and draws the conclusion that the **lines intersect/common point/consistent** or tick. **Note:** If an incorrect value for a is found in part (a) but in part (b) they find that $a = -2$ this scores **B0** but all other marks are available
(c) This is M1M1A1 on ePen marking as M1 A1ft A1 **M1:** Full attempt to find the minimum distance from a point to a plane. Condone a sign slip with the value of d. **A1ft:** Following through on their point of intersection. Uses the model to find a correct expression for minimum distance from the nest to the ground **A1:** Correct distance **Alternative** **M1:** Find the shortest distance from a point to plane by finding the perpendicular distance from the given plane to the origin and the perpendicular distance from the plane contacting their point of intersection to the origin and subtracts **A1ft:** Following through on their point of intersection. Uses the model to find a correct expression for minimum distance from the nest to the ground **A1:** Correct distance
(d) **B1:** Comments on one of the models • Flight path of the birds modelled as a straight line • Angle between flight paths modelled as 120° • The bird's nest is modelled as a point • Ground modelled as a plane Then states unreliabl Any correct answer seen, ignore any other incorrect answers