Question	Scheme	Marks	AOs
9(a)(i)	$\frac{dy}{dx} = \ldots \cosh^{n-1} x \sinh x$ $\frac{d^2y}{dx^2} = \ldots \cosh^{n-2} x \sinh^2 x + \ldots \cosh^{n-1} x \cosh x$ **Alternatively** $y = \left(\frac{e^x+e^{-x}}{2}\right)^n$ leading to $\frac{dy}{dx} = \ldots \left(\frac{e^x+e^{-x}}{2}\right)^{n-1} \left(\frac{e^x-e^{-x}}{2}\right)$ $\frac{d^2y}{dx^2} = \ldots \left(\frac{e^x+e^{-x}}{2}\right)^{n-2} \left(\frac{e^x-e^{-x}}{2}\right)^2 + \ldots \left(\frac{e^x+e^{-x}}{2}\right)^n$	M1	1.1b
	$\frac{dy}{dx} = n \cosh^{n-1} x \sinh x$ $\frac{d^2y}{dx^2} = n(n-1) \cosh^{n-2} x \sinh^2 x + n \cosh^n x$ **Alternatively** $\frac{dy}{dx} = n \left(\frac{e^x+e^{-x}}{2}\right)^{n-1} \left(\frac{e^x-e^{-x}}{2}\right)$ $\frac{d^2y}{dx^2} = n(n-1) \left(\frac{e^x+e^{-x}}{2}\right)^{n-2} \left(\frac{e^x-e^{-x}}{2}\right)^2 + n \left(\frac{e^x+e^{-x}}{2}\right)^n$	A1	2.1
	$\frac{d^2y}{dx^2} = n(n-1) \cosh^{n-2} x (\cosh^2 x - 1) + n \cosh^n x$	M1	2.1
	$\frac{d^2y}{dx^2} = n^2 \cosh^n x - n(n-1) \cosh^{n-2} x$* cso	A1*	1.1b
		(4)	
(a)(ii)	$\frac{d^3y}{dx^3} = \ldots \cosh^{n-1} x \sinh x - \ldots \cosh^{n-3} x \sinh x$ $\frac{d^4y}{dx^4}$ $= \ldots \cosh^{n-2} x \sinh^2 x + \ldots \cosh^n x - \ldots \cosh^{n-4} x \sinh^2 x - \ldots cos$	M1	1.1b
	$\frac{d^3y}{dx^3} = n^3 \cosh^{n-1} x \sinh x - n(n-1)(n-2) \cosh^{n-3} x \sinh x$ $\frac{d^4y}{dx^4} = n^3(n-1) \cosh^{n-2} x \sinh^2 x + n^3 \cosh^n x$ $-n(n-1)(n-2)(n-3) \cosh^{n-4} x \sinh^2 x - n(n-1)(n$ $-2) \cosh^{n-2} x$	A1	1.1b
		(2)	
	Alternative 1 using $\frac{d^2y}{dx^2} = n^2 y - n(n-1) \cosh^{n-2} x$ leading to $\frac{d^3y}{dx^3} = n^2 \frac{dy}{dx} - \ldots \cosh^{n-3} x \sinh x$ $\frac{d^4y}{dx^4} = n^2 \frac{d^2y}{dx^2} - \ldots \cosh^{n-4} x \sinh^2 x - \ldots \cosh^{n-2} x$	M1	1.1b

	$\frac{d^3y}{dx^3} = n^2\frac{dy}{dx} - n(n-1)(n-2)\cosh^{n-3}x\sinh x$ $\frac{d^4y}{dx^4} = n^2\frac{d^2y}{dx^2} - n(n-1)(n-2)(n-3)\cosh^{n-4}x\sinh^2 x - n(n-1)(n-2)\cosh^{n-2}x$	A1	1.1b
		(2)	
	Alternative 2 $y = \cosh^n x \Rightarrow \frac{d^2y}{dx^2} = n^2\cosh^n x - n(n-1)\cosh^{n-2}x$ $y = \cosh^{n-2}x \Rightarrow \frac{d^2y}{dx^2} = \ldots\cosh^{n-2}x - \ldots\cosh^{n-4}x$ $\frac{d^4y}{dx^4} = n^2\left[n^2\cosh^n x - n(n-1)\cosh^{n-2}x\right] - n(n-1)\left[\ldots\cosh^{n-2}x - \ldots\cosh^{n-4}x\right]$	M1	1.1b
	$y = \cosh^n x \Rightarrow \frac{d^2y}{dx^2} = n^2\cosh^n x - n(n-1)\cosh^{n-2}x$ $y = \cosh^{n-2}x \Rightarrow \frac{d^2y}{dx^2} = (n-2)^2\cosh^{n-2}x - (n-2)(n-3)\cosh^{n-4}x$ $\frac{d^4y}{dx^4} = n^2\left[n^2\cosh^n x - n(n-1)\cosh^{n-2}x\right] - n(n-1)\left[(n-2)^2\cosh^{n-2}x - (n-2)(n-3)\cosh^{n-4}x\right]$	A1	1.1b
		(2)	
	Alternative 3 Using $\frac{d^2y}{dx^2} = n^2\left(\frac{e^x+e^{-x}}{2}\right)^n - n(n-1)\left(\frac{e^x+e^{-x}}{2}\right)^{n-2}$ leading to $\frac{d^3y}{dx^3} = \ldots\left(\frac{e^x+e^{-x}}{2}\right)^{n-1}\left(\frac{e^x-e^{-x}}{2}\right) - \ldots\left(\frac{e^x+e^{-x}}{2}\right)^{n-3}\left(\frac{e^x-e^{-x}}{2}\right)$ $\frac{d^4y}{dx^4} = \ldots\left(\frac{e^x+e^{-x}}{2}\right)^{n-2}\left(\frac{e^x-e^{-x}}{2}\right)^2 + \ldots\left(\frac{e^x+e^{-x}}{2}\right)^{n-2}$ $-\ldots\left(\frac{e^x+e^{-x}}{2}\right)^{n-4}\left(\frac{e^x-e^{-x}}{2}\right)^2 - \ldots\left(\frac{e^x+e^{-x}}{2}\right)^{n-2}$	M1	1.1b
	$\frac{d^3y}{dx^3} = n^3\left(\frac{e^x+e^{-x}}{2}\right)^{n-1}\left(\frac{e^x-e^{-x}}{2}\right) - n(n-1)(n-2)\left(\frac{e^x+e^{-x}}{2}\right)^{n-3}\left(\frac{e^x-e^{-x}}{2}\right)$	A1	1.1b

	$\frac{d^4y}{dx^4} = n^3(n-1)\left(\frac{e^x+e^{-x}}{2}\right)^{n-2}\left(\frac{e^x-e^{-x}}{2}\right)^2 + n^3\left(\frac{e^x+e^{-x}}{2}\right)^{n-2}$ $-n(n-1)(n-2)(n-3)\left(\frac{e^x+e^{-x}}{2}\right)^{n-4}\left(\frac{e^x-e^{-x}}{2}\right)^2 - n(n-1)(n-2)\left(\frac{e^x+e^{-x}}{2}\right)^{n-2}$		
		(2)	
(b)	When $x = 0$ $y = 1,\quad y' = 0,\quad y'' = n^2 - n(n-1),\quad y^{(3)} = 0,$ $y^{(4)} = n^3 - n(n-1)(n-2)$ Uses their values in the expansion $y = y(0) + xy'(0) + \frac{x^2}{2!}y''(0) + \frac{x^3}{3!}y^{(3)}(0) + \frac{x^4}{4!}y^{(4)}(0) + \ldots$	M1	1.1b
	$y = 1 + \frac{nx^2}{2} + \frac{(3n^2-2n)x^4}{24} + \ldots$ cso	A1	2.5
		(2)	

(8 marks)

Notes:

(a)(i)

M1: Uses the chain rule and product rule to find the first and second derivatives which must be of the required form, condone sign slips

Alternatively uses the exponential definition and uses the chain rule and product rule to find the first and second derivatives which must be of the required form.

A1: Correct unsimplified first and second derivatives, may be in exponential form.

M1: Uses the identity $\pm \cosh^2 x \pm \sinh^2 x = 1$

A1*: Achieves the printed answer with no errors or omissions e.g. missing x's

(a)(ii)

M1: Uses the chain rule and product rule to find the third and fourth derivatives which must be of the required form, condone sign slips

A1: Correct fourth derivative, does not need to be simplified ISW

Alternative 1

M1: Using $\frac{d^2y}{dx^2} = n^2y - n(n-1)\cosh^{n-2} x$ to find the third and fourth derivatives which must be of the required form, condone sign slips

A1: Correct fourth derivative, does not need to be simplified ISW

Alternative 2

M1: Using $y = \cosh^n x \Rightarrow \frac{d^2y}{dx^2} = n^2 \cosh^n x - n(n-1)\cosh^{n-2} x$

$y = \cosh^{n-2} x \Rightarrow \frac{d^2y}{dx^2} = \ldots\cosh^{n-2} x - \ldots\cosh^{n-4} x$ leading to

$$\frac{d^4y}{dx^4} = n^2[n^2 \cosh^n x - n(n-1)\cosh^{n-2} x] - n(n-1)\left[\text{their}\frac{d(\cosh^{n-2} x)}{dx}\right]$$

A1: Correct fourth derivative, does not need to be simplified ISW

Alternative 3

M1: Uses the exponential definition and uses the chain rule and product rule to find the third and fourth derivatives which must be of the required form.

A1: Correct fourth derivative, does not need to be simplified ISW

(b)

M1: Attempts the evaluation of all four of their derivatives at $x = 0$ and applies the Maclaurin formula with their values. Note that $y^{(1)}(0) = 0$ and $y^{(3)}(0) = 0$ may be implied as they will have a multiple of sinh0. If their $y^{(3)}(0) \neq 0$ they allow this mark for their first 3 non-zero terms

A1: Correct simplified expansion from correct derivatives cso