

Question	Scheme	Marks	AOs
<b>3(a)</b>	$\mathbf{M}^2 + 11\mathbf{M} = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} \Rightarrow \begin{pmatrix} 34 & 5k-10 \\ 6k-12 & k^2+30 \end{pmatrix} + \begin{pmatrix} -22 & 55 \\ 66 & 11k \end{pmatrix} = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix}$	<b>M1</b>	1.1b
	$\Rightarrow a = 12$	<b>A1</b>	2.2a
	$5k - 10 + 55 = 0 \Rightarrow 5k = -45 \Rightarrow k = -9^*$ or $6k - 12 + 66 = 0 \Rightarrow 6k = -54 \Rightarrow k = -9^*$ or $k^2 + 11k + 30 = 12 \Rightarrow k^2 + 11k + 18 = 0 \Rightarrow k = -2, -9^*$ , $k \neq -2$ as $4 \times -2 - 10 + 55 \neq 0$	<b>A1*</b>	2.1
		<b>(3)</b>	
<b>(b)</b>	$\begin{pmatrix} -2 & 5 \\ 6 & -9 \end{pmatrix} \begin{pmatrix} x \\ mx+c \end{pmatrix} = \begin{pmatrix} X \\ mX+c \end{pmatrix} \Rightarrow \begin{cases} -2x+5(mx+c) = X \\ 6x-9(mx+c) = mX+c \end{cases}$	<b>M1</b>	1.1b
	$\Rightarrow 6x - 9mx - 9c = -2mx + 5m^2x + 5mc + c$	<b>M1</b>	3.1a
	$\Rightarrow (5m^2 + 7m - 6)x + (5m + 10)c = 0$	<b>A1</b>	1.1b
	$\Rightarrow 5m^2 + 7m - 6 = 0 \Rightarrow (m+2)(5m-3) \Rightarrow m = -2, \frac{3}{5}$	<b>M1</b>	1.1b
	$m = \frac{3}{5} \Rightarrow 5m + 10 \neq 0$ so need $c = 0$ hence $y = \frac{3}{5}x$ is a fixed line	<b>A1</b>	2.2a
	$m = -2 \Rightarrow 5m + 10 = 0$ so $c$ can be anything, so $y = -2x + c$ for any $c$ is fixed.	<b>A1</b>	2.2a
		<b>(6)</b>	
<b>(c)</b>	$((0, c) \rightarrow (5c, -9c)$ so need $c = 0$ ), $(1, m) \rightarrow (-2 + 5m, 6 - 9m)$ so need $5m = 3$ hence $y = \frac{3}{5}x$ contains fixed points.	<b>B1</b>	3.2a
		<b>(1)</b>	

**(10 marks)**

**Notes:**

**(a)**

**M1:** Evaluates  $\mathbf{M}^2$  and uses in the equation given.

**A1:** Correct value of  $a$  deduced from upper left entries.

**A1\*:** Correct work to show  $k = -9$ . If off diagonals are used no further justification is needed (they are “given” the result is true). If the bottom right entry is used there must be a valid reason for rejecting  $-2$  as a solution (ie checking the off diagonal).

**(b)**

**M1:** Sets up the matrix equation for invariant lines and extracts the simultaneous equations from the matrix equation.

**M1:** Eliminates “ $X$ ” to get a linear equation in “ $x$ ”.

**A1:** Correct equation.

**M1:** Solves the equation in  $m$  by any valid means.

**A1:** Deduces  $y = \frac{3}{5}x$  is a fixed line (where  $c = 0$ ). If the value for  $m$  here is wrong, allow this A for  $y = -2x$  if the general case for the final A is not scored.

**A1:** Deduces  $y = -2x + c$  is a fixed line where  $c$  can be any value. Must include all the lines.

**(c)**

**B1:** Identifies  $y = \frac{3}{5}x$  is a line of fixed points with reason. Allow if  $c = 0$  is assumed. See scheme for one possible reason, others may be given.