Question	Scheme	Marks	AOs
5(a)	$\alpha = \frac{z_1 + z_2}{2} = \frac{35 - 25\mathrm{i} - 29 + 39\mathrm{i}}{2} = \ldots$	M1	1.1b
	$= 3 + 7\mathrm{i}$ *	A1*	1.1b
		(2)	
(b)	$\beta(z_1 - \alpha) = \left(\frac{1+\mathrm{i}}{64}\right)(35 - 25\mathrm{i} - (3 + 7\mathrm{i})) = \left(\frac{1+\mathrm{i}}{64}\right)(32 - 32\mathrm{i}) =$ $= \frac{1}{64}(32 - 32\mathrm{i} + 32\mathrm{i} - 32\mathrm{i}^2) = \frac{1}{64}(32 - 32\mathrm{i} + 32\mathrm{i} + 32)$	M1	1.1b
	$= \frac{1}{64}(64) = 1$*	A1*	1.1b
		(2)	
(c)	Roots are $1\mathrm{e}^{\mathrm{i}\frac{k\pi}{3}}, \quad k = 0, 1, 2, 3, 4, 5$	B1	1.1b
		(1)	
(d)	$w = \beta(z - \alpha) = \mathrm{e}^{\mathrm{i}\frac{k\pi}{3}} \Rightarrow z = \frac{\mathrm{e}^{\mathrm{i}\frac{k\pi}{3}}}{\beta} + \alpha$	M1	3.1a
	$\Rightarrow z = \frac{64\left(\cos\frac{k\pi}{3} + \mathrm{i}\sin\frac{k\pi}{3}\right)(1 - \mathrm{i})}{(1+\mathrm{i})(1-\mathrm{i})} + 3 + 7\mathrm{i} = \ldots$	M1	1.1b
	Two of $19 + 16\sqrt{3} + \left(16\sqrt{3} - 9\right)\mathrm{i},\ 16\sqrt{3} - 13 + \left(23 + 16\sqrt{3}\right)\mathrm{i}$, $-13 - 16\sqrt{3} + \left(23 - 16\sqrt{3}\right)\mathrm{i},\ 19 - 16\sqrt{3} - \left(9 + 16\sqrt{3}\right)\mathrm{i}$	A1	2.5
	All four of $19 + 16\sqrt{3} + \left(16\sqrt{3} - 9\right)\mathrm{i},\ 16\sqrt{3} - 13 + \left(23 + 16\sqrt{3}\right)\mathrm{i}$, $-13 - 16\sqrt{3} + \left(23 - 16\sqrt{3}\right)\mathrm{i},\ 19 - 16\sqrt{3} - \left(9 + 16\sqrt{3}\right)\mathrm{i}$	A1	2.2a
		(4)	
		(11 marks)	

Notes:

(a)

M1: Attempts the midpoint of z_1 and z_2

A1*: Correct point.

(b)

M1: Substitutes into the equation with z_1 and α and β, simplifies and expands and applies $i^2 = -1$

A1*: Correct answer.

(c)

B1: Correct roots, accept all 6 listed or given in general form as in the scheme. Need not show the 1.

(d)

M1: Realises the need to set the roots of unity equal to $\beta(z-\alpha)$ and solve for *z*. Must be attempted at least once with any of their roots.

M1: Finds the Cartesian form from their equation for at least one of the roots other than z_1 and z_2

A1: At least two correct other roots than z_1 and z_2 in Cartesian form.

A1: Deduces all four correct in Cartesian form.