Question	Scheme	Marks	AOs
8(a)	One possibility is (that all three roots lie on) the real axis.	**B1**	2.2a
	The other possibility is that all three roots have the same real part so lie on a vertical line.	**B1**	3.1a
		(2)	
(b)	Other roots are $\frac{3}{2}$ and $\frac{3}{2}-\frac{3}{2}\text{i}$	**B1**	3.2a
		(1)	
(c)(i)	Common root must be $\frac{3}{2}$	**B1**	2.2a
		(1)	
(ii)	So $\text{g}(z)=\left(z-\frac{3}{2}\right)(z+4)(z+\alpha)$	**M1**	1.1b
	$-\frac{3}{2}\times4\times\alpha=12\Rightarrow\alpha=-2$	**M1** **A1**	3.1a 1.1b
		(3)	
(d)	$\text{f}(z)=8\left(z-\frac{3}{2}\right)\left(z-\frac{3}{2}-\frac{3}{2}\text{i}\right)\left(z-\frac{3}{2}+\frac{3}{2}\text{i}\right)=8\left(z-\frac{3}{2}\right)\left(z^2-3z+\frac{9}{2}\right)$	**M1**	1.1b
	$\text{f}(z)=\text{g}(z)\Rightarrow8\left(z-\frac{3}{2}\right)\left(z^2-3z+\frac{9}{2}\right)=\left(z-\frac{3}{2}\right)(z+4)(z-2)$ $\Rightarrow8z^2-24z+36=(z+4)(z-2)\quad\left(\text{or } z=\frac{3}{2}\right)$	**M1**	3.1a
	$\Rightarrow7z^2-26z+44\Rightarrow z=\frac{26\pm\sqrt{26^2-4\times7\times44}}{14}=\ldots$	**M1**	1.1b
	So solutions are $\frac{3}{2}$, $\frac{13\pm\text{i}\sqrt{139}}{7}$	**A1**	1.1b
		(4)	
		(11 marks)	

Notes:

(a)

B1: Identifies the case that all three roots could be real so lie on the real axis. Accept as a diagram or equation given. Mention of real axis is sufficient.

B1: Identifies the case of vertical lines (where there is a complex conjugate pair and the third root must lie on the real axis with same real part as the pair).

(b)

B1: Interprets the conclusion from (a) in context by identifying the correct two roots.

(c)

B1: Deduces the real root is the one in common.

M1: Forms an expression for g(z) using the two known roots. Alternatively, if product of roots is used first, it is for forming a cubic expression for g(z) from their roots.

M1: Uses the given form for g(z) in the question to find the third root/value for α in their expression. May be scored before the previous M if product of roots is used to find the third root before forming the cubic.

A1: Correct expression for g(z), either factorised or expanded.

(d)

M1: Uses their roots of f(z) to form a cubic expression for f(z), and expands to at least a linear term times a quadratic with real coefficients (which may be seen later).

M1: Sets their expressions equal and factorises out or cancels the common term to achieve a quadratic expression in z. Allow if f(z) is not yet expanded.

M1: Expands, gathers terms and solves the resulting quadratic. Allow this mark if the $z=\frac{3}{2}$ solution is not given.

A1: All three correct solutions given.