Question	Scheme	Marks	AOs
9(a)	$\frac{d^2y}{dt^2}=0.032\frac{dx}{dt}-0.025\frac{dy}{dt}$ oe e.g. $\frac{dx}{dt}=\frac{1}{0.032}\left(\frac{d^2y}{dt^2}+0.025\frac{dy}{dt}\right)$	B1	1.1b
	$\frac{d^2y}{dt^2}=0.032(0.025y-0.045x+2)-0.025\frac{dy}{dt}$ $=0.0008y-\frac{0.00144}{0.032}\left(\frac{dy}{dt}+0.025y\right)+0.064-0.025\frac{dy}{dt}$ Or $\frac{1}{0.032}\left(\frac{d^2y}{dt^2}+0.025\frac{dy}{dt}\right)=0.025y-\frac{0.045}{0.032}\left(\frac{dy}{dt}+0.025y\right)+2$	M1	1.1b
	$40\,000\frac{d^2y}{dt^2}+2800\frac{dy}{dt}+13y=2560$ *	A1*	2.1
		(3)	
(b)	$40000m^2+2800m+13=0\Rightarrow m=...$	M1	3.4
	$\text{CF}: y=Ae^{m_1t}+Be^{m_2t}$	M1	1.1b
	$\text{CF}: y=Ae^{\frac{-t}{200}}+Be^{\frac{-13t}{200}}$	A1	1.1b
	PI: Try $y=k\Rightarrow 13k=2560\Rightarrow k=\frac{2560}{13}$	M1	3.4
	GS: $y=Ae^{\frac{-t}{200}}+Be^{\frac{-13t}{200}}+\frac{2560}{13}$	A1ft	1.1b
		(5)	
(c)	$t=0, y=0\Rightarrow 0=A+B+\frac{2560}{13}$	M1	3.4
	$t=0, y=0, x=0\Rightarrow\frac{dy}{dt}=0.032\times0-0.025\times0=0$	B1	3.4
	$\Rightarrow\frac{dy}{dt}=-\frac{A}{200}e^{\frac{-t}{200}}-\frac{13B}{200}e^{\frac{-13t}{200}}=0\Rightarrow-\frac{A}{200}-\frac{13B}{200}=0\Rightarrow A=-13B$	M1	1.1b
	$y=-\frac{640}{3}e^{\frac{-t}{200}}+\frac{640}{39}e^{\frac{-13t}{200}}+\frac{2560}{13}$	A1	1.1b
		(4)	
(d)	As $t\to\infty, e^{-kt}\to 0$ for $k>0$ so $y\to...,$	M1	1.1b

	$y \to \frac{2560}{13} \approx 196.92$ so the rate of administration is sufficient to reach the required level.	**A1**	3.2b
		(2)	
			(14 marks)

Notes:

(a)

B1: Differentiates the second equation with respect to *t* correctly. May have rearranged to make *x* the subject first. The dot notation for derivatives may be used.

M1: Uses the second equation to eliminate *x* to achieve an equation in *y*, $\frac{dy}{dt}, \frac{d^2y}{dt^2}$.

A1*: Achieves the printed answer with no errors.

(b)

M1: Uses the model to form and attempt to solve the auxiliary equation (Accept a correct equation followed by two values for *m* as an attempt to solve.)

M1: Forms the complementary function correct for their roots (so if repeated or complex roots found, award for appropriate form for CF). Must be in terms of *t* only (not *x*)

A1: Correct CF

M1: Chooses the correct form of the PI according to the model and uses a complete method to find the PI

A1ft: Combines their CF (which need not be correct) with the correct PI to give *y* in terms of *t* so look for *y* = their CF + $\frac{2560}{13}$, accepting awrt 197.

(c)

M1: Uses the initial conditions of the model to set up an equation in *A* and *B* from their general solution.

B1: Uses the initial conditions of the model to find the value of $\frac{dy}{dt}$ when $t = 0$

M1: Differentiates their general solution and substitutes $t = 0$ to form another equation in *A* and *B* and proceed at least as far as finding *A* in terms of *B* oe.

A1: Correct particular solution.

(d)

M1: Uses the limit of the exponential terms is zero to find the long term limit of the concentration

A1: Correct limit and concludes the rate of administration is sufficient to achieve the required level.