

5. The points representing the complex numbers  $z_1 = 35 - 25i$  and  $z_2 = -29 + 39i$  are opposite vertices of a regular hexagon,  $H$ , in the complex plane.

The centre of  $H$  represents the complex number  $\alpha$

- (a) Show that  $\alpha = 3 + 7i$

(2)

Given that  $\beta = \frac{1+i}{64}$

- (b) show that

$$\beta(z_1 - \alpha) = 1$$

(2)

The vertices of  $H$  are given by the roots of the equation

$$(\beta(z - \alpha))^6 = 1$$

- (c) (i) Write down the roots of the equation  $w^6 = 1$  in the form  $re^{i\theta}$

(1)

- (ii) Hence, or otherwise, determine the position of the other four vertices of  $H$ , giving your answers as complex numbers in Cartesian form.

(4)