Question	Scheme	Marks	AOs
1(a)	$4\sinh^3 x+3\sinh x\equiv 4\left(\dfrac{e^x-e^{-x}}{2}\right)^3+3\left(\dfrac{e^x-e^{-x}}{2}\right)$ $\equiv 4\left(\dfrac{e^{3x}-3e^x+3e^{-x}-e^{-3x}}{8}\right)+3\left(\dfrac{e^x-e^{-x}}{2}\right)$	M1	2.1
	$\equiv\dfrac{e^{3x}-e^{-3x}}{2}\equiv\sinh 3x$*	A1*	1.1b
		(2)	
(b)	$\sinh 3x=19\sinh x\Rightarrow 4\sinh^3 x+3\sinh x=19\sinh x$ $\sinh 3x=19\sinh x\Rightarrow 4\sinh^3 x-16\sinh x=0$ $4\sinh x\left(\sinh^2 x-4\right)=0$	M1	3.1a
	$\sinh x=0\Rightarrow x=0$	B1	2.2a
	$\sinh^2 x=4\Rightarrow\sinh x=\pm 2$ $\Rightarrow x=\ln\left(\pm 2+\sqrt{(\pm 2)^2+1}\right)$	M1	1.1b
	$x=\ln\left(2+\sqrt{5}\right)$ **or** $x=\ln\left(-2+\sqrt{5}\right)$ oe e.g. $x=-\ln\left(2+\sqrt{5}\right)$	A1	1.1b
	$x=\ln\left(2+\sqrt{5}\right)$ **and** $x=\ln\left(-2+\sqrt{5}\right)$ Alternatively, $x=\ln\left(\sqrt{5}\pm 2\right)$ oe e.g. $x=\pm\ln\left(2+\sqrt{5}\right)$ or $\dfrac{1}{2}\ln\left(9\pm 4\sqrt{5}\right)$	A1	1.1b
		(5)	
			(7 marks)

Notes

(a)

M1: Begins the proof by expressing sinh x correctly in terms of exponentials, substitutes and makes progress in cubing the bracket.

Award for obtaining an expression of the form $Ae^{3x}+Be^x+Ce^{-x}+De^{-3x}$ but terms do not need to be collected.

Note that $(e^{2x}-2+e^{-2x})(e^x-e^{-x}) = e^{3x}-e^x-2e^x+2e^{-x}+e^{-x}-e^{-3x}$

A1*: Fully correct proof with no errors. Must see $=\sinh 3x$ or e.g. LHS = RHS

(b)

M1: Uses part (a), collects terms and attempts to factorise or cancel sinh x.

This can be implied if they go straight from their cubic to writing **all** correct answers for sinh x including zero from their calculator.

B1: Deduces the root x = 0, allow ln1 but not $\ln\left[0+\ln\sqrt{0+1}\right]$

M1: Proceeds to sinh $x = \alpha$ and uses the correct logarithmic form of arsinh to obtain at least one exact value for x

Alternatively, candidates proceed from sinh $x = \alpha$ to substitute the exponential form and then solve a 3TQ in e^x to obtain at least one exact value for x

A1: One correct non-zero solution

A1: Both correct non-zero solutions and no incorrect other solutions, but isw if they go on to evaluate these answers incorrectly.

Allow $x=\ln\left(\sqrt{5}\pm 2\right)$ for listing both solutions

Alternative if candidates substitute the exponential form at the start:

M1: Candidates substitutes the exponential form for each term and proceeds to find a four term cubic equation in e^{2x} = 0, which may not be correct.

B1: Deduces the root x = 0, allow ln1 but not $\ln\left[0+\ln\sqrt{0+1}\right]$

M1: They factorise their cubic, proceed to obtain exact values for e^{2x}, then take logs to obtain at least one exact value for x

If they go directly to decimal answers this will usually score M0 unless they recover to exact form.

A1: One correct non-zero solution

A1: Both correct non-zero solutions and no incorrect other solutions, but isw if they go on to evaluate these answers incorrectly.

This is how this would be awarded:

$$\frac{(e^{3x}-e^{-3x})}{2}=19\left(\frac{e^{x}-e^{-x}}{2}\right)$$

$$e^{3x}-e^{-3x}-19e^{x}+19e^{-x}=0$$

$$e^{6x}-1-19e^{4x}+19e^{2x}=0$$ M1 (for cubic in e^{2x} = 0)

$$e^{6x}+19e^{4x}+19e^{2x}-1=0$$

$$\Rightarrow(e^{2x}-1)(e^{4x}-18e^{2x}+1)=0$$

$$e^{2x}=1,\ 9\pm4\sqrt{5}$$

$$2x=\ln1,\ 2x=\ln\left(9\pm4\sqrt{5}\right)$$

$$x=0,\ x=\frac{1}{2}\ln\left(9\pm4\sqrt{5}\right)$$ B1, M1, A1, A1