Question	Scheme	Marks	AOs
2(a)	$\dfrac{\mathrm{d}\left(\dfrac{3-x}{6+x}\right)}{\mathrm{d}x}=\dfrac{-(6+x)-(3-x)}{(6+x)^2}$	M1 A1	3.1a 1.1b
	$\mathrm{f}(x)=\tanh^{-1}\left(\dfrac{3-x}{6+x}\right)\Rightarrow \mathrm{f}'(x)=\dfrac{1}{1-\left(\dfrac{3-x}{6+x}\right)^2}\times\dfrac{-9}{(6+x)^2}$	dM1	3.1a
	$=\dfrac{(6+x)^2}{36+12x+x^2-9+6x-x^2}\times\dfrac{-9}{(6+x)^2}=\dfrac{-9}{18x+27}=\dfrac{-1}{2x+3}$ *	A1*	2.1
		(4)	
	Alternative 1 for part (a)		
	$\dfrac{\mathrm{d}\left(\dfrac{3-x}{6+x}\right)}{\mathrm{d}x}=\dfrac{-(6+x)-(3-x)}{(6+x)^2}$	M1 A1	3.1a 1.1b
	$y=\tanh^{-1}\left(\dfrac{3-x}{6+x}\right)\Rightarrow \tanh y=\dfrac{3-x}{6+x}\Rightarrow \operatorname{sech}^2 y\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{-9}{(6+x)^2}$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{1-\tanh^2 y}\times\dfrac{-9}{(6+x)^2}=\dfrac{1}{1-\left(\dfrac{3-x}{6+x}\right)^2}\times\dfrac{-9}{(6+x)^2}$	dM1	3.1a
	$=\dfrac{(6+x)^2}{36+12x+x^2-9+6x-x^2}\times\dfrac{-9}{(6+x)^2}=\dfrac{-9}{18x+27}=\dfrac{-1}{2x+3}$ *	A1*	2.1
		(4)	
	Alternative 2 for part (a)		
	$\mathrm{f}(x)=\tanh^{-1}\left(\dfrac{3-x}{6+x}\right)=\dfrac{1}{2}\ln\left(\dfrac{1+\dfrac{3-x}{6+x}}{1-\dfrac{3-x}{6+x}}\right)=\dfrac{1}{2}\ln\left(\dfrac{9}{3+2x}\right)$	M1 A1	3.1a 1.1b
	$\mathrm{f}(x)=\tanh^{-1}\left(\dfrac{3-x}{6+x}\right)\Rightarrow \mathrm{f}'(x)=\dfrac{1}{2}\times\dfrac{3+2x}{9}\times\dfrac{-18}{(3+2x)^2}$	dM1	3.1a
	$=\dfrac{-1}{2x+3}$ *	A1*	1.1b
		(4)	
	Alternative 3 for part (a)		
	$\tanh y=\mathrm{f}(x)=\left(\dfrac{3-x}{6+x}\right)\Rightarrow\dfrac{\mathrm{e}^y-\mathrm{e}^{-y}}{\mathrm{e}^y+\mathrm{e}^{-y}}=\dfrac{3-x}{6+x}$ or $\dfrac{\mathrm{e}^{2y}-1}{\mathrm{e}^{2y}+1}=\dfrac{3-x}{6+x}$ $\mathrm{e}^{2y}=\dfrac{9}{2x+3}$	M1 A1	3.1a 1.1b
	$y=\dfrac{1}{2}\ln\left(\dfrac{9}{2x+3}\right)\Rightarrow \mathrm{f}'(x)=\dfrac{1}{2}\times\dfrac{2x+3}{9}\times\dfrac{-18}{(2x+3)^2}$	dM1	3.1a
	$=\dfrac{-1}{2x+3}$ *	A1	1.1b
		(4)	

Notes

(a) **Do not allow misreads of tan*x* for tanh*x* in this question; this would be a maximum of M1A1 for part (a)**

M1: Attempts to use the quotient (or product) rule on $\frac{3-x}{6+x}$ to obtain an expression of the form

$$\frac{A(6+x)-B(3-x)}{(6+x)^2},\ B>0 \qquad \textbf{or} \qquad C(3-x)(6+x)^{-2}+D(6+x)^{-1}$$

Alternatively, candidates may also write $\frac{3-x}{6+x}$ as $-1+\frac{9}{6+x}$ and then differentiate to find an expression of the form $\frac{E}{(6+x)^2}$ which may be seen embedded in their working.

They may also write $\frac{3-x}{6+x}$ as $\frac{3}{6+x}-\frac{x}{6+x}$ and then differentiate to find an expression of the form $\frac{E}{(6+x)^2}$ oe

A1: Correct expression in any form.

dM1: A complete method to find the derivative using the chain rule to obtain

$$\frac{1}{1-\left(\frac{3-x}{6+x}\right)^2}\times\left(\text{their } \frac{-9}{(6+x)^2}\right)$$

This is dependent on the first M mark.

A1*: Reaches the printed answer with sufficient working and no errors, with at least one intermediate line.

Alternative 1:
M1: see main scheme
A1: see main scheme

dM1: A complete method to find the derivative using the chain rule:
Rearranges the equation and uses implicit differentiation. Must proceed from $\text{sech}^2 y$ or equivalent to $1-\tanh^2 y$ and then substitute for $\tanh y$
This is dependent on the first M mark.
A1*: see main scheme

Alternative 2:

M1: Uses the logarithmic form of artanh to obtain $k\ln\left(\frac{1+\frac{3-x}{6+x}}{1-\frac{3-x}{6+x}}\right)$

A1: Correct simplified expression

dM1: A complete method to find the derivative to obtain

$$\text{f}'(x)=k\times\left(1\div\left(\text{their}\frac{9}{3+2x}\right)\right)\times\left(\text{their } \frac{9}{3+2x}\right)^2$$

Alternatively uses log rules to partition their expression and then differentiates each term.
This is dependent on the first M mark.

A1*: Reaches the printed answer with sufficient working and no errors.
Alternative 3:
M1: Takes tanh of both sides to obtain tanh*y* in terms of *x* and expresses tanh*y* correctly in terms of exponentials and makes e^{2y} the subject.
A1: A correct expression for e^{2y}

dM1: Rearranges their function to the form $k\ln\left(\frac{a}{bx+c}\right)$ **and** uses the chain rule to find a derivative of the form $f'(x)=\frac{1}{2}\times\frac{bx+c}{a}\times\frac{k}{(bx+c)^2}$ or uses log rules to partition their expression and then differentiates each term.
This is dependent on the first M mark.

A1*: Reaches the printed answer with sufficient working and no errors

(b)	$\left[f''(x)=\right]\ \frac{2}{(2x+3)^2}$	B1	1.1b
		(1)	
(c)	$f(0)=\tanh^{-1}\left(\frac{1}{2}\right)\left(=\frac{1}{2}\ln 3\right),\ f'(0)=-\frac{1}{3},\ f''(0)=\frac{2}{9}$	M1	1.1b
	$\left[f(x)\right]=f(0)+xf'(0)+\frac{x^2}{2}f''(0)$	M1	1.1b
	$=\ln\sqrt{3}-\frac{1}{3}x+\frac{1}{9}x^2$	A1	1.1b
		(3)	
			(8 marks)

Notes

(b)
B1: Correct second derivative in any form such as e.g. $\frac{2}{(4x^2+12x+9)}$ or $2(2x+3)^{-2}$

(c)
M1: Attempts at least two of the values of f(0), f′(0) and f″(0)
M1: Correct application of the Maclaurin series for **all** of $f(0)$, $f'(0)$ and $f''(0)$ where all are non-zero values. Substitutes their values into a correct expression. This mark is not dependent.

A1: Correct expansion.
Award if written as a single expression, or *p*, *q* and *r* written separately.
Ignore any extra terms written as powers of x^3 and above, isw when a correct answer is seen.