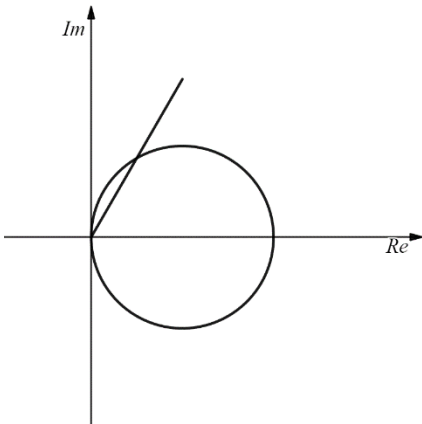
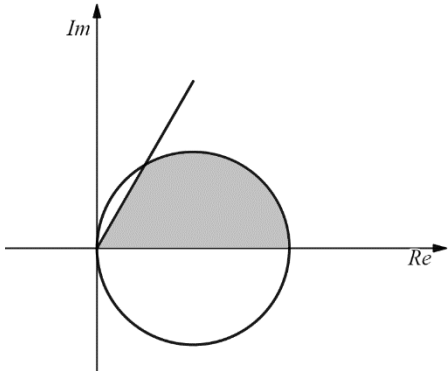
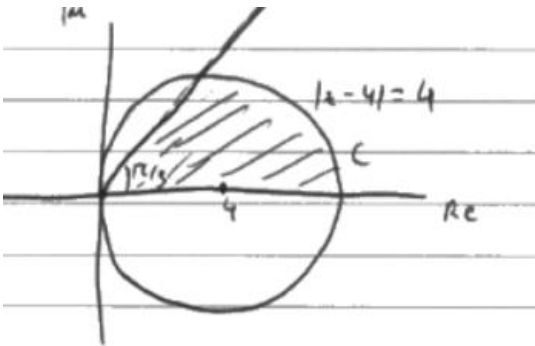


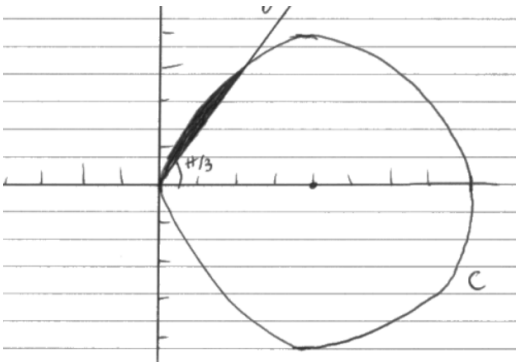
Question	Scheme	Marks	AOs
5(a)		M1	1.1b
		A1	1.1b
		M1	1.1b
		A1	1.1b
		(4)	
(b)		B1ft	2.3
		(1)	

Notes			
<p>(a)</p> <p>M1: Circle drawn with centre on the real axis Look for real axis acting as a line of symmetry of the circle.</p> <p>A1: Circle in the correct position with the imaginary axis as a tangent Centre need not be labelled for either mark.</p> <p>M1: Half line starting at the origin, must be in the first quadrant. Do not award if their line continues into the third quadrant</p> <p>A1: Fully correct diagram that requires</p> <ul style="list-style-type: none"> • A circle in the correct position • A half line intersecting the circle at the origin and in the first quadrant • The x coordinate of the intersection in the first quadrant must be to the left of the centre of the circle <p>(b)</p> <p>B1ft: Shades the region in their circle above the real axis and below the half line For this mark to be awarded their line must intersect the circle.</p>			

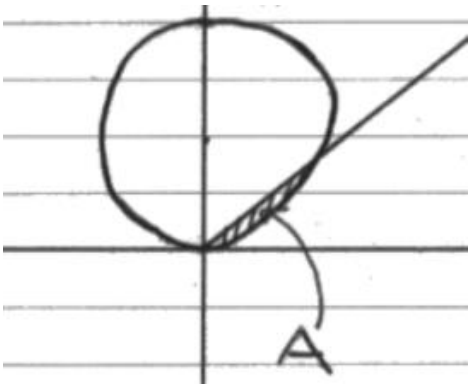
Q5 (a) and (b) examples



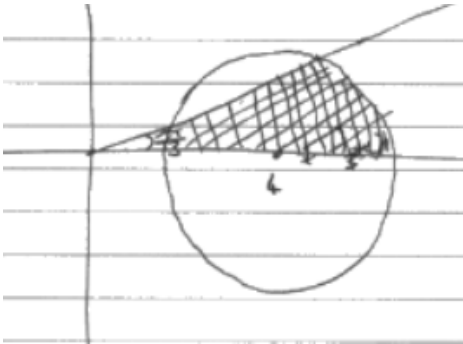
a) M1A1M1A1 b) B1



a) M1A1M1A1 b) B0



a) M0A0M1A0 b) B1



a) M1A0M1A0 b) B1

(c)	$(x-4)^2 + y^2 = 16, \quad y = \sqrt{3}x \Rightarrow x^2 - 8x + 16 + 3x^2 = 16 \Rightarrow x = \dots$ $x = 2, \quad y = 2\sqrt{3}$	M1 A1	3.1a 1.1b
	$\frac{1}{2}\pi \times 4^2 - \left(\frac{1}{2} \times 4^2 \times \frac{\pi}{3} - \frac{1}{2} \times 4^2 \times \sin \frac{\pi}{3} \right)$ <p>or</p> $\frac{1}{2} \times 4^2 \times \frac{2\pi}{3} + \frac{1}{2} \times 4 \times 2\sqrt{3}$ <p>or</p> $\frac{1}{2} \times 4^2 \times \frac{2\pi}{3} + \frac{1}{2} \times 4 \times 4 \times \frac{\sqrt{3}}{2}$	dM1	3.1a
	$= \frac{16}{3}\pi + 4\sqrt{3}$	A1	1.1b
		(4)	
	Alternative 1 for part (c)		
	$x = 4\cos \frac{\pi}{3}, \quad y = 4\sin \frac{\pi}{3}$	M1 A1	3.1a 1.1b
	$\frac{1}{2}\pi \times 4^2 - \left(\frac{1}{2} \times 4^2 \times \frac{\pi}{3} - \frac{1}{2} \times 4^2 \times \sin \frac{\pi}{3} \right)$ <p>or</p> $\frac{1}{2} \times 4^2 \times \frac{2\pi}{3} + \frac{1}{2} \times 4 \times 2\sqrt{3}$ <p>or</p> $\frac{1}{2} \times 4^2 \times \frac{2\pi}{3} + \frac{1}{2} \times 4 \times 4 \times \frac{\sqrt{3}}{2}$	dM1	3.1a
	$= \frac{16}{3}\pi + 4\sqrt{3}$	A1	1.1b
		(4)	
	Alternative 2 for part (c)		
	<p>Deduces $\frac{\pi}{3}$ means there is an equilateral triangle of length 4</p> <p>4 and $\sin \frac{\pi}{3}$ oe are seen or used in their workings</p>	M1 A1	3.1a 1.1b
	$\frac{1}{2}\pi \times 4^2 - \left(\frac{1}{2} \times 4^2 \times \frac{\pi}{3} - \frac{1}{2} \times 4^2 \times \sin \frac{\pi}{3} \right)$ <p>or</p> $\frac{1}{2} \times 4^2 \times \frac{2\pi}{3} + \frac{1}{2} \times 4 \times 2\sqrt{3}$ <p>or</p>	dM1	3.1a

	$\frac{1}{2} \times 4^2 \times \frac{2\pi}{3} + \frac{1}{2} \times 4 \times 4 \times \frac{\sqrt{3}}{2}$		
	$= \frac{16}{3} \pi + 4\sqrt{3}$	A1	1.1b
		(4)	
	Alternative 3 for part (c) Polar coordinates		
	$(x-4)^2 + y^2 = 16, \Rightarrow r^2 = 8r \cos \theta \Rightarrow r = 8 \cos \theta$ $\text{Area} = \int \frac{1}{2} r^2 \, d\theta = \int \frac{1}{2} \cdot 64 \cos^2 \theta \, d\theta = \int (16 + 16 \cos 2\theta) \, d\theta$	M1 A1	3.1a 1.1b
	$= [16\theta + 8 \sin 2\theta]_0^{\frac{\pi}{3}} = \dots$	dM1	3.1a
	$= \frac{16}{3} \pi + 4\sqrt{3}$	A1	1.1b
		(4)	

(9 marks)

Notes

(c)

This is not for finding their shaded area of their diagram in part (a) but is for a correct process for finding the correct required area.

M1: Correct strategy for identifying both coordinates of the point of intersection.

Award for substituting a line of the form $y = kx$ into the equation of a circle $(x-4)^2 + y^2 = 16$ then proceeds to find a value for x from a quadratic, where $x \neq 0$ and then find their y coordinate.

A1: Correct coordinates, either written separately, or as $(2, 2\sqrt{3})$ or $2 + 2\sqrt{3}i$

dM1: Fully correct strategy for the area, must be consistent with a half line making an angle of $\frac{\pi}{3}$ with the real axis.

Can be found by subtracting the area of a segment from the area of a semicircle or by adding the area of a sector to the area of a triangle.

The candidate may do combinations of semicircles, triangles and sectors so look carefully for the method.

A1: Correct answer in the required form.

Alternative 1

M1: Deduces that since the half line makes an angle of $\frac{\pi}{3}$ with the real axis, the horizontal and vertical distances from the origin to the point of intersection are $4\cos\frac{\pi}{3}$ and $4\sin\frac{\pi}{3}$.

A1: $4\cos\frac{\pi}{3}$ and $4\sin\frac{\pi}{3}$ oe are seen or used in their workings

dM1: Fully correct strategy for the area, must be consistent with a half line making an angle of $\frac{\pi}{3}$ with the real axis.

Can be found by subtracting the area of a segment from the area of a semicircle or by adding the area of a sector to the area of a triangle.

The candidate may do combinations of semicircles, triangles and sectors so look carefully for the method.

A1: Correct answer in the required form.

Alternative 2

M1: Alternatively deduces that the angle implies that there is an equilateral triangle of radius 4

A1: 4 and $\sin \frac{\pi}{3}$ are seen or used in their workings

dm1: Fully correct strategy for the area, must be consistent with a half line making an angle of $\frac{\pi}{3}$ with the real axis.

Can be found by subtracting the area of a segment from the area of a semicircle or by adding the area of a sector to the area of a triangle.

The candidate may do combinations of semicircles, triangles and sectors so look carefully for the method.

A1: Correct answer in the required form.

Alternative 3: using polar coordinates

M1: Achieves a polar equation of the form $r = k \cos \theta$ and uses $\left(\frac{1}{2}\right) \int r^2 d\theta$ to obtain $k \int \cos^2 \theta d\theta$

A1: Obtains $\int (16 + 16 \cos 2\theta) d\theta$ oe, $d\theta$ may be missing

dm1: Integrates to obtain an expression of the form $a\theta + b \sin 2\theta$, substitutes in limits of 0 and $\frac{\pi}{3}$ and subtracts. If they reach a correct answer with no integration seen then withhold this mark.

A1: Correct answer in the form required

$$\text{e.g. } (x-4)^2 + y^2 = 16, \Rightarrow r^2 = 8r \cos \theta \Rightarrow r = 8 \cos \theta$$

$$\text{Area} = \int \frac{1}{2} r^2 d\theta = \int \frac{1}{2} \cdot 64 \cos^2 \theta d\theta = \int \frac{32(1 + \cos 2\theta)}{2} d\theta = \int (16 + 16 \cos 2\theta) d\theta = [16\theta + 8 \sin 2\theta]$$

$$[16\theta + 8 \sin 2\theta]_0^{\frac{\pi}{3}} = \left[16\left(\frac{\pi}{3}\right) + 8 \sin 2\left(\frac{\pi}{3}\right) \right] - [0] = \frac{16}{3} \pi + 4\sqrt{3}$$