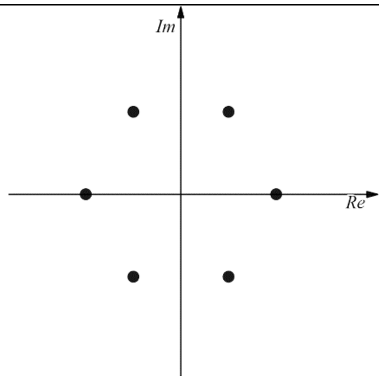
Question	Scheme	Marks	AOs
7(a)	$z = e^{\frac{k\pi}{3}i}, \quad k = 0, 1, 2, 3, 4, 5$	M1 A1	1.1b 1.1b
		(2)	
(b)	 	B1 dB1	2.2a 1.1b
		(2)	
(c)	e.g. $\left(\sqrt{3}+i\right)^6 = \left(2e^{\frac{\pi}{6}i}\right)^6 = 64e^{i\pi} = -64*$ **or** $\left[2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)\right]^6 = 2^6(\cos\pi + i\sin\pi) = 64(-1) = -64*$ **or** $\left(\sqrt{3}+i\right)^6 = \left(\sqrt{3}\right)^6 + 6\left(\sqrt{3}\right)^5 i - 15\left(\sqrt{3}\right)^4 - 20\left(\sqrt{3}\right)^3 i + 15\left(\sqrt{3}\right)^2 + 6\sqrt{3}i + i^6$ $= 27 - 135 + 45 - 1 = -64*$ **or** $\left(\sqrt{3}+i\right)^6 = = 27 + 54\sqrt{3}i + 135i^2 + 60\sqrt{3}i^3 + 45i^4 + 6\sqrt{3}i^5 + i^6$ $= 27 + 54\sqrt{3}i - 135 - 60\sqrt{3}i + 45 + 6\sqrt{3}i - 1 = -64*$	M1 A1*	1.1b 2.1
		(2)	
(d)	$r = 2$	B1	2.2a
	$z = 2e^{\frac{\pi}{6}i} \times e^{\frac{k\pi}{3}i}, \quad k = 0, 1, 2, 3, 4, 5$	M1	3.1a
	$z = 2e^{\left(\frac{\pi}{6}+\frac{k\pi}{3}\right)i}, \quad k = 0, 1, 2, 3, 4, 5$	A1	1.1b
		(3)	
			(9 marks)

Notes

(a)

M1: For sight of $e^{\frac{k\pi}{3}i}$
Accept any value for k

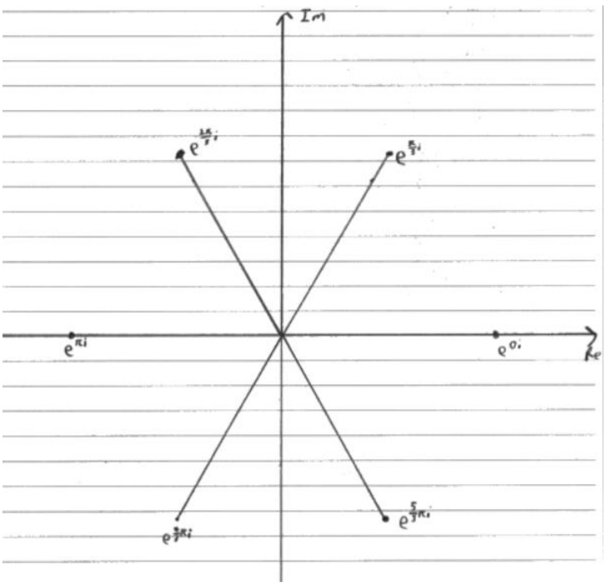
A1: All six roots fully defined as shown or listed separately with their values of θ within the given range with no incorrect or extra values. Ensure i and π are present in each term.

Note: Roots if listed are $e^0, e^{\frac{\pi}{3}i}, e^{\frac{2\pi}{3}i}, e^{\pi i}, e^{\frac{4\pi}{3}i}, e^{\frac{5\pi}{3}i}$, condone 1 for e^0 and/or -1 for $e^{\pi i}$

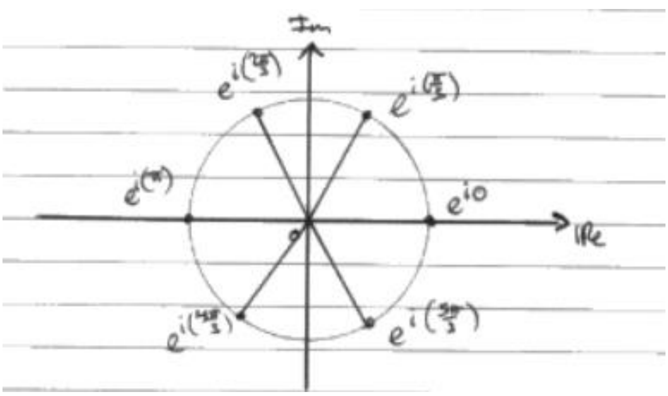
(b)

B1: Plots 6 points that form a hexagon, with a point on the positive real axis and a point on the negative real axis, and one point in each quadrant.
Do not be concerned about the position of each point from the centre, however the sketch must convey a hexagon.
dB1: The points form a hexagon, centre the origin (see diagram), axes need not be labelled.
Look for the axes acting as lines of symmetry.
(Drawing line/vectors to each point is acceptable but not necessary for either mark)

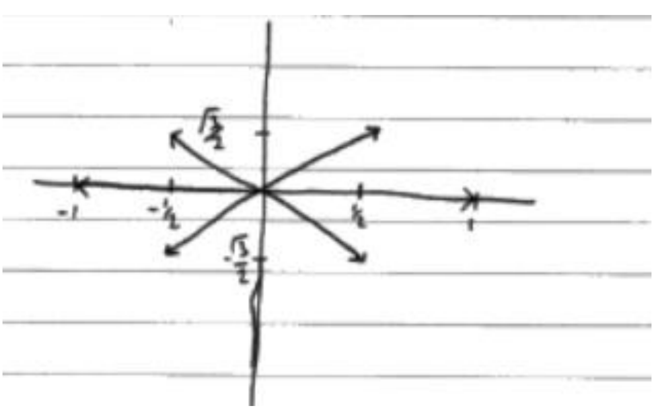
Examples



B1dB1



B1dB1



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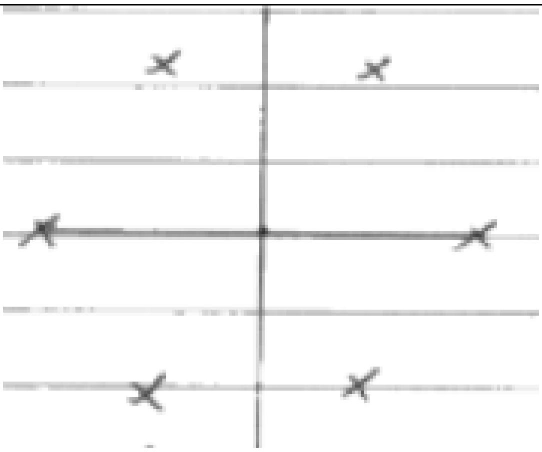

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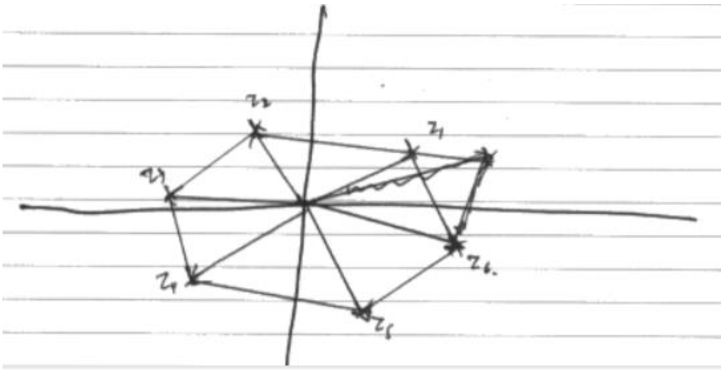
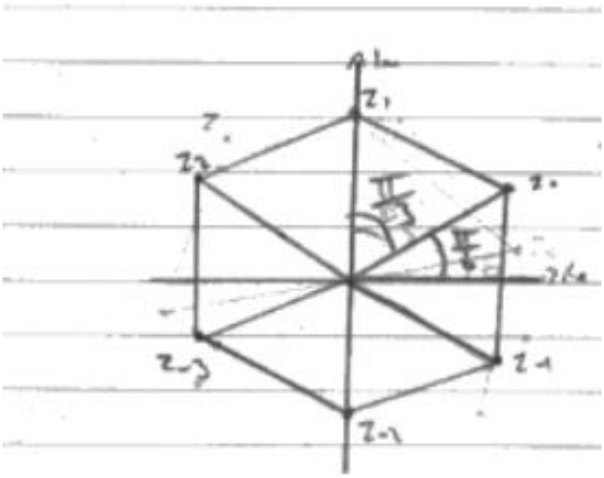


B0dB0



B0dB0

(c)

M1: Converts $\sqrt{3}+\mathrm{i}$ to polar form to obtain $r\mathrm{e}^{\mathrm{i}\theta}$ with at least r = 2 or $\theta = \dfrac{\pi}{6}$ and applies the power of 6 correctly to obtain $r^6\mathrm{e}^{6\theta\mathrm{i}}$

A1*: Obtains the given answer with sufficient working shown.

As a minimum need to see $2^6\mathrm{e}^{\frac{6\pi i}{6}} = -64$ or $2^6\mathrm{e}^{\pi\mathrm{i}} = -64$

If r = −2 is seen in their workings withhold this mark.

OR

M1: Converts $\sqrt{3}+\mathrm{i}$ to modulus-argument form $r(\cos\theta + \mathrm{i}\sin\theta)$ with at least r = 2 or $\theta = \dfrac{\pi}{6}$ and applies the power of 6 correctly to obtain $r^6(\cos 6\theta + \mathrm{i}\sin 6\theta)$

A1*: Obtains the given answer with sufficient working shown.

OR

M1: Attempts to expand $\left(\sqrt{3}+\mathrm{i}\right)^6$ fully using an attempt at the binomial expansion. Must have 7 terms for $(a+b)^n$ and correct binomial coefficients with $a = \sqrt{3}$, $b = \mathrm{i}$ and $n = 6$

A1*: Obtains the given answer with at least one intermediate line.

OR

M1: Attempts the full expansion of $\left(\sqrt{3}+\mathrm{i}\right)^6 = (\sqrt{3}+\mathrm{i})(\sqrt{3}+\mathrm{i})(\sqrt{3}+\mathrm{i})\ldots(\sqrt{3}+\mathrm{i}) =$

There must be no brackets, no irrational numbers and no terms in i in their simplified answer.

A1*: Obtains the given answer with sufficient working shown including correct full expansion, with at least one intermediate line.

(d)

B1: Deduces $r = 2$ (only)

M1: Obtains at least one value of z in the form $r\mathrm{e}^{\mathrm{i}\theta}$ with their consistent value of r, and θ taking one of

$$\left\{\frac{\pi}{6},\frac{\pi}{2},\frac{5\pi}{6},\frac{7\pi}{6},\frac{3\pi}{2},\frac{11\pi}{6}\right\}$$

A1: For $2\mathrm{e}^{\frac{\pi}{6}\mathrm{i}}, 2\mathrm{e}^{\frac{\pi}{2}\mathrm{i}}, 2\mathrm{e}^{\frac{5\pi}{6}\mathrm{i}}, 2\mathrm{e}^{\frac{7\pi}{6}\mathrm{i}}, 2\mathrm{e}^{\frac{3\pi}{2}\mathrm{i}}, 2\mathrm{e}^{\frac{11\pi}{6}\mathrm{i}}$ with no incorrect or extra values. Accept unsimplified arguments such as having a solution of $2\mathrm{e}^{\frac{9\pi}{6}\mathrm{i}}$. Ensure i and π are present in each term.

Accept $2\mathrm{e}^{\frac{\pi}{2}\mathrm{i}}$ as $2\mathrm{i}$ and $2\mathrm{e}^{\frac{3\pi}{2}\mathrm{i}}$ as $-2\mathrm{i}$