6. The motion of a particle P along the x-axis is modelled by the differential equation

$$2\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 5\frac{\mathrm{d}x}{\mathrm{d}t} + 2x = 4t + 12$$

where P is x metres from the origin O at time t seconds, $t \geqslant 0$

(a) Determine the general solution of the differential equation.

(6)

(b) Hence determine the particular solution for which $x = 3$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = -2$ when $t = 0$

(3)

(c) (i) Show that, according to the model, the minimum distance between O and P is $(2 + \ln 2)$ metres.

(ii) Justify that this distance is a minimum.

(4)

For large values of t the particle is expected to move with constant speed.

(d) Comment on the suitability of the model in light of this information.

(1)