

Question	Scheme	Marks	AOs
10	Use of $\frac{\sin(\theta+h) - \sin \theta}{(\theta+h) - \theta}$	B1	2.1
	Uses the compound angle identity for $\sin(A+B)$ with $A = \theta, B = h$ $\Rightarrow \sin(\theta+h) = \sin \theta \cos h + \cos \theta \sin h$	M1	1.1b
	Achieves $\frac{\sin(\theta+h) - \sin \theta}{h} = \frac{\sin \theta \cos h + \cos \theta \sin h - \sin \theta}{h}$	A1	1.1b
	$= \frac{\sin h}{h} \cos \theta + \left( \frac{\cos h - 1}{h} \right) \sin \theta$	M1	2.1
	Uses $h \rightarrow 0, \frac{\sin h}{h} \rightarrow 1$ and $\frac{\cos h - 1}{h} \rightarrow 0$ Hence the $\lim_{h \rightarrow 0} \frac{\sin(\theta+h) - \sin \theta}{(\theta+h) - \theta} = \cos \theta$ and the gradient of the chord $\rightarrow$ gradient of the curve $\Rightarrow \frac{dy}{d\theta} = \cos \theta$ *	A1*	2.5

(5 marks)

**Notes:**

**B1:** States or implies that the gradient of the chord is  $\frac{\sin(\theta+h) - \sin \theta}{h}$  or similar such as

$$\frac{\sin(\theta + \delta\theta) - \sin \theta}{\theta + \delta\theta - \theta} \text{ for a small } h \text{ or } \delta\theta$$

**M1:** Uses the compound angle identity for  $\sin(A+B)$  with  $A = \theta, B = h$  or  $\delta\theta$

**A1:** Obtains  $\frac{\sin \theta \cos h + \cos \theta \sin h - \sin \theta}{h}$  or equivalent

**M1:** Writes their expression in terms of  $\frac{\sin h}{h}$  and  $\frac{\cos h - 1}{h}$

**A1\*:** Uses correct language to explain that  $\frac{dy}{d\theta} = \cos \theta$

For this method they should use all of the given statements  $h \rightarrow 0, \frac{\sin h}{h} \rightarrow 1,$

$$\frac{\cos h - 1}{h} \rightarrow 0 \text{ meaning that the } \lim_{h \rightarrow 0} \frac{\sin(\theta+h) - \sin \theta}{(\theta+h) - \theta} = \cos \theta$$

and therefore the gradient of the chord  $\rightarrow$  gradient of the curve  $\Rightarrow \frac{dy}{d\theta} = \cos \theta$

Question	Scheme	Marks	AOs
<b>10alt</b>	Use of $\frac{\sin(\theta+h) - \sin \theta}{(\theta+h) - \theta}$	B1	2.1
	Sets $\frac{\sin(\theta+h) - \sin \theta}{(\theta+h) - \theta} = \frac{\sin\left(\theta + \frac{h}{2} + \frac{h}{2}\right) - \sin\left(\theta + \frac{h}{2} - \frac{h}{2}\right)}{h}$ and uses the compound angle identity for $\sin(A+B)$ and $\sin(A-B)$ with $A = \theta + \frac{h}{2}$ , $B = \frac{h}{2}$	M1	1.1b
	Achieves $\frac{\sin(\theta+h) - \sin \theta}{h} =$ $\frac{\left[\sin\left(\theta + \frac{h}{2}\right)\cos\left(\frac{h}{2}\right) + \cos\left(\theta + \frac{h}{2}\right)\sin\left(\frac{h}{2}\right)\right] - \left[\sin\left(\theta + \frac{h}{2}\right)\cos\left(\frac{h}{2}\right) - \cos\left(\theta + \frac{h}{2}\right)\sin\left(\frac{h}{2}\right)\right]}{h}$	A1	1.1b
	$= \frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}} \times \cos\left(\theta + \frac{h}{2}\right)$	M1	2.1
	Uses $h \rightarrow 0$ , $\frac{h}{2} \rightarrow 0$ hence $\frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}} \rightarrow 1$ and $\cos\left(\theta + \frac{h}{2}\right) \rightarrow \cos \theta$ Therefore the $\lim_{h \rightarrow 0} \frac{\sin(\theta+h) - \sin \theta}{(\theta+h) - \theta} = \cos \theta$ and the gradient of the chord $\rightarrow$ gradient of the curve $\Rightarrow \frac{dy}{d\theta} = \cos \theta$ *	A1*	2.5

(5 marks)

**Additional notes:**

**A1\*:** Uses correct language to explain that  $\frac{dy}{d\theta} = \cos \theta$ . For this method they should use the

(adapted) given statement  $h \rightarrow 0$ ,  $\frac{h}{2} \rightarrow 0$  hence  $\frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}} \rightarrow 1$  with  $\cos\left(\theta + \frac{h}{2}\right) \rightarrow \cos \theta$

meaning that the  $\lim_{h \rightarrow 0} \frac{\sin(\theta+h) - \sin \theta}{(\theta+h) - \theta} = \cos \theta$  and therefore the gradient of the

chord  $\rightarrow$  gradient of the curve  $\Rightarrow \frac{dy}{d\theta} = \cos \theta$