

Question	Scheme	Marks	AOs
<b>10 (a)</b>	$y = \frac{3x-5}{x+1} \Rightarrow y(x+1) = 3x-5 \Rightarrow xy + y = 3x-5 \Rightarrow y+5 = 3x-xy$	M1	1.1b
	$\Rightarrow y+5 = x(3-y) \Rightarrow \frac{y+5}{3-y} = x$	M1	2.1
	Hence $f^{-1}(x) = \frac{x+5}{3-x}, \quad x \in \mathbb{R}, x \neq 3$	A1	2.5
		<b>(3)</b>	
<b>(b)</b>	$ff(x) = \frac{3\left(\frac{3x-5}{x+1}\right) - 5}{\left(\frac{3x-5}{x+1}\right) + 1}$	M1	1.1a
	$\frac{3(3x-5) - 5(x+1)}{x+1}$	M1	1.1b
	$= \frac{(3x-5) + (x+1)}{x+1}$	A1	1.1b
	$= \frac{9x-15-5x-5}{3x-5+x+1} = \frac{4x-20}{4x-4} = \frac{x-5}{x-1}$ (note that $a = -5$ )	A1	2.1
		<b>(4)</b>	
<b>(c)</b>	$fg(2) = f(4-6) = f(-2) = \frac{3(-2)-5}{-2+1}; = 11$	M1	1.1b
		A1	1.1b
		<b>(2)</b>	
<b>(d)</b>	$g(x) = x^2 - 3x = (x-1.5)^2 - 2.25$ . Hence $g_{\min} = -2.25$	M1	2.1
	Either $g_{\min} = -2.25$ or $g(x) \geq -2.25$ or $g(5) = 25 - 15 = 10$	B1	1.1b
	$-2.25 \leq g(x) \leq 10$ or $-2.25 \leq y \leq 10$	A1	1.1b
		<b>(3)</b>	
<b>(e)</b>	E.g. <ul style="list-style-type: none"> <li>the function <math>g</math> is many-one</li> <li>the function <math>g</math> is not one-one</li> <li>the inverse is one-many</li> <li><math>g(0) = g(3) = 0</math></li> </ul>	B1	2.4
		<b>(1)</b>	
<b>(13 marks)</b>			

**Question 10 Notes:****(a)****M1:** Attempts to find the inverse by cross-multiplying and an attempt to collect all the  $x$ -terms (or swapped  $y$ -terms) onto one side**M1:** A fully correct method to find the inverse**A1:** A correct  $f^{-1}(x) = \frac{x+5}{3-x}$ ,  $x \in \mathbb{R}$ ,  $x \neq 3$ , expressed fully in function notation (including the domain)**(b)****M1:** Attempts to substitute  $f(x) = \frac{3x-5}{x+1}$  into  $\frac{3f(x)-5}{f(x)+1}$ **M1:** Applies a method of “rationalising the denominator” for both their numerator and their denominator.**A1:** 
$$\frac{3(3x-5) - 5(x+1)}{\frac{x+1}{(3x-5) + (x+1)}} \text{ which can be simplified or un-simplified}$$
$$\frac{x+1}{x+1}$$
**A1:** Shows  $ff(x) = \frac{x+a}{x-1}$  where  $a = -5$  or  $ff(x) = \frac{x-5}{x-1}$ , with no errors seen.**(c)****M1:** Attempts to substitute the result of **g(2)** into  $f$ **A1:** Correctly obtains  $fg(2) = 11$ **(d)****M1:** Full method to establish the minimum of  $g$ .

E.g.

- $(x \pm \alpha)^2 + \beta$  leading to  $g_{\min} = \beta$
- Finds the value of  $x$  for which  $g'(x) = 0$  and inserts this value of  $x$  back into  $g(x)$  in order to find to  $g_{\min}$

**B1:** For either

- finding the correct minimum value of  $g$   
(Can be implied by  $g(x) \geq -2.25$  or  $g(x) > -2.25$ )
- stating  $g(5) = 25 - 15 = 10$

**A1:** States the correct range for  $g$ . E.g.  $-2.25 \leq g(x) \leq 10$  or  $-2.25 \leq y \leq 10$ **(e)****B1:** See scheme