Question	Scheme	Marks	AOs
12	$\int_0^{\frac{\pi}{2}} \frac{\sin 2\theta}{1+\cos\theta}\, d\theta$		
	Attempts this question by applying the substitution $u = 1+\cos\theta$ and progresses as far as achieving $\int \dots \frac{(u-1)}{u} \dots$	M1	3.1a
	$u = 1+\cos\theta \Rightarrow \frac{du}{d\theta} = -\sin\theta$ and $\sin 2\theta = 2\sin\theta\cos\theta$	M1	1.1b
	$\left\{\int \frac{\sin 2\theta}{1+\cos\theta}\, d\theta = \right\} \int \frac{2\sin\theta\cos\theta}{1+\cos\theta}\, d\theta = \int \frac{-2(u-1)}{u}\, du$	A1	2.1
	$-2\int\left(1-\frac{1}{u}\right) du = -2(u - \ln u)$	M1	1.1b
		M1	1.1b
	$\left\{\int_0^{\frac{\pi}{2}} \frac{\sin 2\theta}{1+\cos\theta}\, d\theta = \right\} = -2\left[\, u - \ln u \,\right]_2^1 = -2((1-\ln 1)-(2-\ln 2))$	M1	1.1b
	$= -2(-1+\ln 2) = 2 - 2\ln 2$ *	A1*	2.1
		(7)	
12 Alt 1	Attempts this question by applying the substitution $u = \cos\theta$ and progresses as far as achieving $\int \dots \frac{u}{u+1} \dots$	M1	3.1a
	$u = \cos\theta \Rightarrow \frac{du}{d\theta} = -\sin\theta$ and $\sin 2\theta = 2\sin\theta\cos\theta$	M1	1.1b
	$\left\{\int \frac{\sin 2\theta}{1+\cos\theta}\, d\theta = \right\} \int \frac{2\sin\theta\cos\theta}{1+\cos\theta}\, d\theta = \int \frac{-2u}{u+1}\, du$	A1	2.1
	$\left\{= -2\int \frac{(u+1)-1}{u+1}\, du = -2\int 1 - \frac{1}{u+1}\, du\right\} = -2(u - \ln(u+1))$	M1	1.1b
		M1	1.1b
	$\left\{\int_0^{\frac{\pi}{2}} \frac{\sin 2\theta}{1+\cos\theta}\, d\theta = \right\} = -2\left[\, u - \ln(u+1) \,\right]_1^0 = -2((0-\ln 1)-(1-\ln 2))$	M1	1.1b
	$= -2(-1+\ln 2) = 2 - 2\ln 2$ *	A1*	2.1
		(7)	
		(7 marks)	

Question 12 Notes:	
M1:	See scheme
M1:	Attempts to differentiate $u = 1+\cos\theta$ to give $\frac{du}{d\theta} = ...$ and applies $\sin 2\theta = 2\sin\theta\cos\theta$
A1:	Applies $u = 1+\cos\theta$ to show that the integral becomes $\int \frac{-2(u-1)}{u}\,du$
M1:	Achieves an expression in u that can be directly integrated (e.g. dividing each term by u or applying partial fractions) and integrates to give an expression in u of the form $\pm\lambda u \pm \mu\ln u,\ \lambda,\mu \neq 0$
M1:	For integration in u of the form $\pm 2(u - \ln u)$
M1:	Applies u-limits of 1 and 2 to an expression of the form $\pm\lambda u \pm \mu\ln u,\ \lambda,\mu \neq 0$ and subtracts either way round
A1*:	Applies u-limits the right way round, i.e. • $\int_2^1 \frac{-2(u-1)}{u}\,du = -2\int_2^1 \left(1-\frac{1}{u}\right)du = -2\left[u - \ln u\right]_2^1 = -2((1-\ln 1)-(2-\ln 2))$ • $\int_2^1 \frac{-2(u-1)}{u}\,du = 2\int_1^2 \left(1-\frac{1}{u}\right)du = 2\left[u - \ln u\right]_1^2 = 2((2-\ln 2)-(1-\ln 1))$ and correctly proves $\int_0^{\frac{\pi}{2}} \frac{\sin 2\theta}{1+\cos\theta}\,d\theta = 2-2\ln 2$, with no errors seen
Alt 1	
M1:	See scheme
M1:	Attempts to differentiate $u = \cos\theta$ to give $\frac{du}{d\theta} = ...$ and applies $\sin 2\theta = 2\sin\theta\cos\theta$
A1:	Applies $u = \cos\theta$ to show that the integral becomes $\int \frac{-2u}{u+1}\,du$
M1:	Achieves an expression in u that can be directly integrated (e.g. by applying partial fractions or a substitution $v = u+1$) and integrates to give an expression in u of the form $\pm\lambda u \pm \mu\ln(u+1),\ \lambda,\mu \neq 0$ or $\pm\lambda v \pm \mu\ln v,\ \lambda,\mu \neq 0$, where $v = u+1$
M1:	For integration in u in the form $\pm 2(u - \ln(u+1))$
M1:	Either • Applies u-limits of 0 and 1 to an expression of the form $\pm\lambda u \pm \mu\ln(u+1),\ \lambda,\mu \neq 0$ and subtracts either way round • Applies v-limits of 1 and 2 to an expression of the form $\pm\lambda v \pm \mu\ln v,\ \lambda,\mu \neq 0$, where $v = u+1$ and subtracts either way round
A1*:	Applies u-limits the right way round, (o.e. in v) i.e. • $\int_1^0 \frac{-2u}{u+1}\,du = -2\int_1^0 \left(1-\frac{1}{u+1}\right)du = -2\left[u - \ln(u+1)\right]_1^0 = -2((0-\ln 1)-(1-\ln 2))$ • $\int_1^0 \frac{-2u}{u+1}\,du = 2\int_0^1 \left(1-\frac{1}{u+1}\right)du = 2\left[u - \ln(u+1)\right]_0^1 = 2((1-\ln 2)-(0-\ln 1))$ and correctly proves $\int_0^{\frac{\pi}{2}} \frac{\sin 2\theta}{1+\cos\theta}\,d\theta = 2-2\ln 2$, with no errors seen