

13. (a) Express $2 \sin \theta - 1.5 \cos \theta$ in the form $R \sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

State the value of R and give the value of α to 4 decimal places.

(3)

Tom models the depth of water, D metres, at Southview harbour on 18th October 2017 by the formula

$$D = 6 + 2 \sin\left(\frac{4\pi t}{25}\right) - 1.5 \cos\left(\frac{4\pi t}{25}\right), \quad 0 \leq t \leq 24,$$

where t is the time, in hours, after 00:00 hours on 18th October 2017.

Use Tom's model to

- (b) find the depth of water at 00:00 hours on 18th October 2017,

(1)

- (c) find the maximum depth of water,

(1)

- (d) find the time, in the afternoon, when the maximum depth of water occurs.

Give your answer to the nearest minute.

(3)

Tom's model is supported by measurements of D taken at regular intervals on 18th October 2017. Jolene attempts to use a similar model in order to model the depth of water at Southview harbour on 19th October 2017.

Jolene models the depth of water, H metres, at Southview harbour on 19th October 2017 by the formula

$$H = 6 + 2 \sin\left(\frac{4\pi x}{25}\right) - 1.5 \cos\left(\frac{4\pi x}{25}\right), \quad 0 \leq x \leq 24,$$

where x is the time, in hours, after 00:00 hours on 19th October 2017.

By considering the depth of water at 00:00 hours on 19th October 2017 for both models,

- (e) (i) explain why Jolene's model is not correct,

(ii) hence find a suitable model for H in terms of x .

(3)

(Total for Question 13 is 11 marks)