Question	Scheme	Marks	AOs
9(a)	Either $3y^2 \to Ay\dfrac{\mathrm{d}y}{\mathrm{d}x}$ or $2xy \to 2x\dfrac{\mathrm{d}y}{\mathrm{d}x}+2y$	M1	2.1
	$2x-2x\dfrac{\mathrm{d}y}{\mathrm{d}x}-2y+6y\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$	A1	1.1b
	$(6y-2x)\dfrac{\mathrm{d}y}{\mathrm{d}x}=2y-2x$	M1	2.1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{2y-2x}{6y-2x}=\dfrac{y-x}{3y-x}$ *	A1*	1.1b
		(4)	
(b)	$\left(\text{At } P \text{ and } Q\ \dfrac{\mathrm{d}y}{\mathrm{d}x}\to\infty\Rightarrow\right)$ Deduces that $3y-x=0$	M1	2.2a
	Solves $y=\dfrac{1}{3}x$ and $x^2-2xy+3y^2=50$ simultaneously	M1	3.1a
	$\Rightarrow x=(\pm)5\sqrt{3}$ OR $\Rightarrow y=(\pm)\dfrac{5}{3}\sqrt{3}$	A1	1.1b
	Using $y=\dfrac{1}{3}x\Rightarrow x=..$ AND $y=..$	dM1	1.1b
	$P=\left(-5\sqrt{3},-\dfrac{5}{3}\sqrt{3}\right)$	A1	2.2a
		(5)	
(c)	Explains that you need to solve $y=x$ and $x^2-2xy+3y^2=50$ simultaneously and choose the positive solution	B1ft	2.4
		(1)	
			(10 marks)

Notes:

(a)

M1: For selecting the appropriate method of differentiating either $3y^2 \to Ay\dfrac{\mathrm{d}y}{\mathrm{d}x}$ or $2xy \to 2x\dfrac{\mathrm{d}y}{\mathrm{d}x}+2y$

It may be quite difficult awarding it for the product rule but condone $-2xy \to -2x\dfrac{\mathrm{d}y}{\mathrm{d}x}+2y$ unless you see evidence that they have used the incorrect law vu'-uv'

A1: Fully correct derivative $2x-2x\dfrac{\mathrm{d}y}{\mathrm{d}x}-2y+6y\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$

Allow attempts where candidates write $2x\mathrm{d}x-2x\mathrm{d}y-2y\mathrm{d}x+6y\mathrm{d}y=0$

but watch for students who write $\dfrac{\mathrm{d}y}{\mathrm{d}x}=2x-2x\dfrac{\mathrm{d}y}{\mathrm{d}x}-2y+6y\dfrac{\mathrm{d}y}{\mathrm{d}x}$ This, on its own, is A0 unless you are convinced that this is just their notation. Eg $\dfrac{\mathrm{d}y}{\mathrm{d}x}=2x-2x\dfrac{\mathrm{d}y}{\mathrm{d}x}-2y+6y\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$

M1: For a valid attempt at making $\frac{dy}{dx}$ the subject. with two terms in $\frac{dy}{dx}$ coming from $3y^2$ and $2xy$

Look for $(...\pm...)\frac{dy}{dx} = ...$ It is implied by $\frac{dy}{dx} = \frac{2y-2x}{6y-2x}$

This cannot be scored from attempts such as $\frac{dy}{dx} = 2x - 2x\frac{dy}{dx} - 2y + 6y$ which only has one correct term.

A1*: $\frac{dy}{dx} = \frac{y-x}{3y-x}$ with no errors or omissions.

The previous line $\frac{dy}{dx} = \frac{2y-2x}{6y-2x}$ or equivalent must be seen.

(b)

M1: Deduces that $3y - x = 0$ oe

M1: Attempts to find either the *x* or *y* coordinates of *P* and *Q* by solving their $y = \frac{1}{3}x$ with $x^2 - 2xy + 3y^2 = 50$ simultaneously. Allow for finding a quadratic equation in *x* or *y* and solving to find at least one value for *x* or *y*.

This may be awarded when candidates make the numerator = 0 ie using $y = x$

A1: $\Rightarrow x = (\pm)5\sqrt{3}$ OR $\Rightarrow y = (\pm)\frac{5}{3}\sqrt{3}$

dM1: Dependent upon the previous M, it is for finding the *y* coordinate from their *x* (or vice versa)

This may also be scored following the numerator being set to 0 ie using $y = x$

A1: Deduces that $P = \left(-5\sqrt{3}, -\frac{5}{3}\sqrt{3}\right)$ OE. Allow to be $x = ...$ $y = ...$

(c)

B1ft: Explains that this is where $\frac{dy}{dx} = 0$ and so you need to solve $y = x$ and $x^2 - 2xy + 3y^2 = 50$ simultaneously and choose the positive solution (or larger solution).

Allow a follow through for candidates who mix up parts (b) and (c)

Alternatively candidates could complete the square $(x-y)^2 + 2y^2 = 50$ and state that *y* would reach a maximum value when $x = y$ and choose the positive solution from $2y^2 = 50$