

| Question | Scheme | Marks | AOs |
|----------|--|------------|------|
| 11 (a) | $\sqrt{\frac{1+4x}{1-x}} = (1+4x)^{0.5} \times (1-x)^{-0.5}$ | B1 | 3.1a |
| | $(1+4x)^{0.5} = 1 + 0.5 \times (4x) + \frac{0.5 \times -0.5}{2} \times (4x)^2$ | M1 | 1.1b |
| | $(1-x)^{-0.5} = 1 + (-0.5)(-x) + \frac{(-0.5) \times (-1.5)}{2} (-x)^2$ | M1 | 1.1b |
| | $(1+4x)^{0.5} = 1 + 2x - 2x^2$ and $(1-x)^{-0.5} = 1 + 0.5x + 0.375x^2$ oe | A1 | 1.1b |
| | $(1+4x)^{0.5} \times (1-x)^{-0.5} = (1+2x-2x^2 \dots) \times \left(1 + \frac{1}{2}x + \frac{3}{8}x^2 \dots\right)$ $= 1 + \frac{1}{2}x + \frac{3}{8}x^2 + 2x + x^2 - 2x^2 + \dots$ $= A + Bx + Cx^2$ | dM1 | 2.1 |
| | $= 1 + \frac{5}{2}x - \frac{5}{8}x^2 \dots$ * | A1* | 1.1b |
| | | (6) | |
| (b) | Expression is valid $ x < \frac{1}{4}$ Should not use $x = \frac{1}{2}$ as $\frac{1}{2} > \frac{1}{4}$ | B1 | 2.3 |
| | | (1) | |
| (c) | Substitutes $x = \frac{1}{11}$ into $\sqrt{\frac{1+4x}{1-x}} \approx 1 + \frac{5}{2}x - \frac{5}{8}x^2$ | M1 | 1.1b |
| | $\sqrt{\frac{3}{2}} = \frac{1183}{968}$ | A1 | 1.1b |
| | (so $\sqrt{6}$ is) $\frac{1183}{484}$ or $\frac{2904}{1183}$ | A1 | 2.1 |
| | | (3) | |

(10 marks)

(a)

B1: Scored for key step in setting up the process so that it can be attempted using binomial expansions

This could be achieved by $\sqrt{\frac{1+4x}{1-x}} = (1+4x)^{0.5} \times (1-x)^{-0.5}$ See end for other alternatives

It may be implied by later work.

M1: Award for an attempt at the binomial expansion $(1+4x)^{0.5} = 1 + 0.5 \times (4x) + \frac{(0.5) \times (-0.5)}{2} \times (4x)^2$

There must be three (or more terms). Allow a missing bracket on the $(4x)^2$ and a sign slip so the correct application may be implied by $1 + 2x \pm 0.5x^2$

M1: Award for an attempt at the binomial expansion $(1-x)^{-0.5} = 1 + (-0.5)(-x) + \frac{(-0.5) \times (-1.5)}{2} (-x)^2$

There must be three (or more terms). Allow a missing bracket on the $(-x)^2$ and a sign slips so the method may be awarded on $1 \pm 0.5x \pm 0.375x^2$

A1: Both correct and simplified. This may be awarded for a correct final answer if a candidate does all their simplification at the end

dM1: In the main scheme it is for multiplying their two expansions to reach a quadratic. It is for the key step in adding 'six' terms to produce the quadratic expression. Higher power terms may be seen. Condone slips on

the multiplication on one term only. It is dependent upon having scored the first B and one of the other two M's

In the alternative it is for multiplying $\left(1 + \frac{5}{2}x - \frac{5}{8}x^2\right)(1-x)^{0.5}$ and comparing it to $(1+4x)^{0.5}$

It is for the key step in adding 'six' terms to produce the quadratic expression.

A1*: Completes proof with no errors or omissions. In the alternative there must be some reference to the fact that both sides are equal.

(b)

B1: States that the expansion may not / is not valid when $|x| > \frac{1}{4}$

This may be implied by a statement such as $\frac{1}{2} > \frac{1}{4}$ or stating that the expansion is only valid when $|x| < \frac{1}{4}$

Condone, for this mark a candidate who substitutes $x = \frac{1}{2}$ into the $4x$ **and** states it is not valid as $2 > 1$ oe

Don't award for candidates who state that $\frac{1}{2}$ is too big without any reference to the validity of the expansion.

As a rule you should see some reference to $\frac{1}{4}$ or $4x$

(c)(i)

M1: Substitutes $x = \frac{1}{11}$ into BOTH sides $\sqrt{\frac{1+4x}{1-x}} \approx 1 + \frac{5}{2}x - \frac{5}{8}x^2$ and attempts to find at least one side.

As the left hand side is $\frac{\sqrt{6}}{2}$ they may multiply by 2 first which is acceptable

A1: Finds both sides leading to a correct equation/statement $\sqrt{\frac{15}{10}} = \frac{1183}{968}$ oe $\sqrt{6} = 2 \times \frac{1183}{968}$

A1: $\sqrt{6} = \frac{1183}{484}$ or $\sqrt{6} = \frac{2904}{1183}$ $\sqrt{6} = 2 \times \frac{1183}{968} = \frac{1183}{484}$ would imply all 3 marks

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Watch for other equally valid alternatives for 11(a) including

B1: $(1+4x)^{0.5} \approx \left(1 + \frac{5}{2}x - \frac{5}{8}x^2\right)(1-x)^{0.5}$ then the M's are for $(1+4x)^{0.5}$ and $(1-x)^{0.5}$

M1: $(1-x)^{0.5} = 1 + (0.5)(-x) + \frac{(0.5) \times (-0.5)}{2}(-x)^2$

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Or

B1: $\sqrt{\frac{1+4x}{1-x}} = \sqrt{1 + \frac{5x}{1-x}} = \left(1 + 5x(1-x)^{-1}\right)^{\frac{1}{2}}$ then the first M1 for one application of binomial and the second would be for both $(1-x)^{-1}$ and $(1-x)^{-2}$

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Or

B1: $\sqrt{\frac{1+4x}{1-x}} \times \frac{\sqrt{1-x}}{\sqrt{1-x}} = \sqrt{(1+3x-4x^2)} \times (1-x)^{-1} = \left(1 + (3x-4x^2)\right)^{\frac{1}{2}} \times (1-x)^{-1}$