Question	Scheme	Marks	AOs
4	$f(x) = \dfrac{12x}{3x+4} \quad x \in \mathbb{R},\ x \geqslant 0$		
(a)	$0 \leqslant f(x) < 4$	M1	1.1b
		A1	1.1b
		(2)	
(b)	$y = \dfrac{12x}{3x+4} \Rightarrow y(3x+4) = 12x \Rightarrow 3xy + 4y = 12x \Rightarrow 4y = 12x - 3xy$	M1	1.1b
	$4y = x(12-3y) \Rightarrow \dfrac{4y}{12-3y} = x$	M1	2.1
	Hence $f^{-1}(x) = \dfrac{4x}{12-3x} \quad 0 \leqslant x < 4$	A1	2.5
		(3)	
(c)	$ff(x) = \dfrac{12\left(\dfrac{12x}{3x+4}\right)}{3\left(\dfrac{12x}{3x+4}\right)+4}$	M1	1.1b
	$= \dfrac{\dfrac{144x}{3x+4}}{\dfrac{36x+12x+16}{3x+4}}$	M1	1.1b
	$= \dfrac{144x}{48x+16} = \dfrac{9x}{3x+1}$ * $\quad \{x \in \mathbb{R},\ x \geqslant 0\}$	A1*	2.1
		(3)	
(d)	$\left\{ff(x) = \dfrac{7}{2} \Rightarrow\right\}\ \dfrac{9x}{3x+1} = \dfrac{7}{2} \Rightarrow 18x = 21x + 7 \Rightarrow -3x = 7 \Rightarrow x = ...$	M1	1.1b
	Reject $x = -\dfrac{7}{3}$ As $ff(x)$ is valid for $x \geqslant 0$, then $ff(x) = \dfrac{7}{2}$ has no solutions	A1	2.4
		(2)	
(d) Alt 1	$\left\{ff(x) = \dfrac{7}{2} \Rightarrow\right\}\ f(x) = f^{-1}\left(\dfrac{7}{2}\right) = \dfrac{4\left(\dfrac{7}{2}\right)}{12-3\left(\dfrac{7}{2}\right)}$	M1	1.1b
	$\{f(x) =\}\ f^{-1}\left(\dfrac{7}{2}\right) = \dfrac{28}{3}$ As $0 \leqslant f(x) < 4$ and as $\dfrac{28}{3} > 4$, then $ff(x) = \dfrac{7}{2}$ has no solutions	A1	2.4
		(2)	
			(10 marks)

Question	Scheme	Marks	AOs
4 (d) Alt 2	Range of $\text{ff}(x)$ is $0 \leqslant \text{ff}(x) < 3$	M1	1.1b
	As $\frac{7}{2} > 3$, then $\text{ff}(x) = \frac{7}{2}$ has no solutions	A1	2.4
		(2)	

Question 4 Notes:

(a)	
M1:	For one "end" fully correct; e.g. accept $\text{f}(x) \geqslant 0$ (**not** $x \geqslant 0$) or $\text{f}(x) < 4$ (**not** $x < 4$); or for both correct "end" values; e.g. accept $0 < \text{f}(x) \leqslant 4$.
A1:	Correct range using correct notation. Accept $0 \leqslant \text{f}(x) < 4$, $0 \leqslant y < 4$, $[0, 4)$, $\text{f}(x) \geqslant 0$ and $\text{f}(x) < 4$
(b)	
M1:	Attempts to find the inverse by cross-multiplying and an attempt to collect all the x-terms (or swapped y-terms) onto one side.
M1:	A fully correct method to find the inverse.
A1:	A correct $\text{f}^{-1}(x) = \frac{4x}{12-3x}$, $0 \leqslant x < 4$, o.e. expressed fully in function notation, including the domain, which may be correct or followed through from their part (a) answer for their range of f
Note:	Writing $y = \frac{12x}{3x+4}$ as $y = \frac{4(3x+4)-16}{3x+4} \Rightarrow y = 4 - \frac{16}{3x+4}$ leads to a correct $\text{f}^{-1}(x) = \frac{1}{3}\left(\frac{16}{4-x} - 4\right)$, $0 \leqslant x < 4$
(c)	
M1:	Attempts to substitute $\text{f}(x) = \frac{12x}{3x+4}$ into $\frac{12\text{f}(x)}{3\text{f}(x)+4}$
M1:	Applies a method of "rationalising the denominator" for their denominator.
A1*:	Shows $\text{ff}(x) = \frac{9x}{3x+1}$ with no errors seen. **Note:** The domain of $\text{ff}(x)$ is not required in this part.
(d)	
M1:	Sets $\frac{9x}{3x+1}$ to $\frac{7}{2}$ and solves to find $x = \ldots$
A1:	Finds $x = -\frac{7}{3}$, rejects this solution as $\text{ff}(x)$ is valid for $x \geqslant 0$ only. Concludes that $\text{ff}(x) = \frac{7}{2}$ has no solutions.

Question 4 Notes Continued:

(d) **Alt 1**	
M1:	Attempts to find $f^{-1}\left(\frac{7}{2}\right)$
A1:	Deduces $f(x) = f^{-1}\left(\frac{7}{2}\right) = \frac{28}{3}$ and concludes $ff(x) = \frac{7}{2}$ has no solutions because $f(x) = \frac{28}{3}$ lies outside the range $0 \leqslant f(x) < 4$
(d) **Alt 2**	
M1:	Evidence that the upper bound of $ff(x)$ is 3
A1:	$0 \leqslant ff(x) < 3$ and concludes that $ff(x) = \frac{7}{2}$ has no solutions because $\frac{7}{2} > 3$