

Question	Scheme	Marks	AOs
<b>6 (a)</b>	$\left\{ u = e^{\frac{1}{2}x} \text{ or } x = 2\ln u \Rightarrow \right\}$		
	$\frac{du}{dx} = \frac{1}{2}e^{\frac{1}{2}x} \text{ or } \frac{du}{dx} = \frac{1}{2}u \text{ or } \frac{dx}{du} = \frac{2}{u} \text{ or } dx = \frac{2}{u}du \text{ or } 2du = udx, \text{ etc.}$	B1	1.1b
	<b>Criteria 1</b> $\left\{ x = 0 \Rightarrow a = e^0 \text{ and } x = 2 \Rightarrow b = e^{\frac{1}{2}(2)} \right\}$ $a = 1, b = e \text{ or evidence of } 0 \rightarrow 1 \text{ and } 2 \rightarrow e$		
	<b>Criteria 2 (dependent on the first B1 mark)</b> $\int \frac{6}{(e^{\frac{1}{2}x} + 4)} dx = \int \frac{6}{(u + 4)} \cdot \frac{2}{u} du = \int \frac{12}{u(u + 4)} du$		
	Either Criteria 1 or Criteria 2	B1	1.1b
	Both Criteria 1 and Criteria 2 <b>and</b> correctly achieves the result $\int_1^e \frac{12}{u(u + 4)} du$	B1	2.1
		<b>(3)</b>	
<b>(b)</b>	$\frac{12}{u(u + 4)} \equiv \frac{A}{u} + \frac{B}{(u + 4)} \Rightarrow 12 \equiv A(u + 4) + Bu$	M1	1.1b
	$u = 0 \Rightarrow A = 3; u = -4 \Rightarrow B = -3$	A1	1.1b
	$\left\{ \int \frac{12}{u(u + 4)} du = \right\} \int \left( \frac{3}{u} - \frac{3}{(u + 4)} \right) du = 3\ln u - 3\ln(u + 4)$	M1	3.1a
		A1ft	1.1b
	$\left\{ \text{So, } [3\ln u - 3\ln(u + 4)]_1^e \right\}$		
	$= (3\ln e - 3\ln(e + 4)) - (3\ln 1 - 3\ln 5)$		
	$= 3\ln e - 3\ln(e + 4) + 3\ln 5$		
	$= 3\ln \left( \frac{5e}{e + 4} \right) *$	A1*	2.1
		<b>(5)</b>	

**(8 marks)**

## Question 6 Notes:

(a)

**B1:** See scheme

**B1:** See scheme

**B1:** See scheme

**Note for Criteria 2:** Must start from one of

- $\int y dx$ , with integral sign and  $dx$
- $\int \frac{6}{e^{\frac{1}{2}x} + 4} dx$ , with integral sign and  $dx$
- $\int \frac{6}{e^{\frac{1}{2}x} + 4} \frac{dx}{du} du$ , with integral sign and  $\frac{dx}{du} du$

and end at  $\int \frac{12}{u(u+4)} du$ , with integral sign and  $du$ , **with no incorrect working**

(b)

**M1:**

Writing  $\frac{12}{u(u+4)} \equiv \frac{A}{u} + \frac{B}{(u+4)}$ , o.e. or  $\frac{1}{u(u+4)} \equiv \frac{P}{u} + \frac{Q}{(u+4)}$ , o.e. and a complete method for finding the values of both **their A** and **their B** (or **their P** and **their Q**)

**Note:** This mark can be implied by writing down  $\frac{"A"}{u} + \frac{"B"}{(u+4)}$  with values stated for **their A** and **their B** where either **their A** = 3 or **their B** = -3

**A1:**

Both **their A** = 3 and **their B** = -3 (or **their P** =  $\frac{1}{4}$  and **their Q** =  $-\frac{1}{4}$  with a factor of 12 in front of the integral sign)

**M1:**

Complete strategy for finding  $\int \frac{12}{u(u+4)} du$ , which consists of

- expressing  $\frac{12}{u(u+4)}$  in partial fractions
- and integrating  $\frac{12}{u(u+4)} \equiv \frac{M}{u} \pm \frac{N}{(u \pm k)}$ ;  $M, N, k \neq 0$ ; (i.e. **a two-term partial fraction**) to obtain **both**  $\pm \lambda \ln(\alpha u)$  **and**  $\pm \mu \ln(\beta(u \pm k))$ ;  $\lambda, \mu, \alpha, \beta \neq 0$

**A1ft:**

Integration of both terms is **correctly followed through** from **their M** and **their N**

**A1\*:**

Applies limits of e and 1 in  $u$  (or applies limits of 2 and 0 in  $x$ ), subtracts the correct way round and uses laws of logarithms to correctly obtain  $3 \ln \left( \frac{5e}{e+4} \right)$  with no errors seen.