E.g. $y^{2}-4 y+7=(y-2)^{2}-4+7$

$$
=(y-2)^{2}+3 \geqslant 3, \text { as }(y-2)^{2} \geqslant 0
$$

and so $y^{2}-4 y+7$ is positive for all real values of $y$
(ii) For an explanation or statement to show when (Bobby's) claim $\mathrm{e}^{3 x} \geqslant \mathrm{e}^{2 x}$ fails. This could be e.g.

- when $x=-1, \mathrm{e}^{-3}<\mathrm{e}^{-2}$ or $\mathrm{e}^{-3}$ is not greater than or equal to $\mathrm{e}^{-2}$
- when $x<0, \mathrm{e}^{3 x}<\mathrm{e}^{2 x}$ or $\mathrm{e}^{3 x}$ is not greater than or equal to $\mathrm{e}^{2 x}$

Followed by an explanation or statement to show when (Bobby's) claim $\mathrm{e}^{3 x} \geqslant \mathrm{e}^{2 x}$ is true. This could be e.g.

- $\boldsymbol{x}=2, \mathrm{e}^{6} \geqslant \mathrm{e}^{4}$ or $\mathrm{e}^{6}$ is greater than or equal to $\mathrm{e}^{4}$
- when $x \geqslant 0, \mathrm{e}^{3 x} \geqslant \mathrm{e}^{2 x}$
and a correct conclusion. E.g.
- (Bobby's) claim is sometimes true
(ii) $\quad$ Assuming $\mathrm{e}^{3 x} \geqslant \mathrm{e}^{2 x}$, then $\ln \left(\mathrm{e}^{3 x}\right) \geqslant \ln \left(\mathrm{e}^{2 x}\right) \Rightarrow 3 x \geqslant 2 x \Rightarrow x \geqslant 0 \quad$ M1 $\quad 2.3$

Correct algebra, using logarithms, leading from $\mathrm{e}^{3 x} \geqslant \mathrm{e}^{2 x}$ to $x \geqslant 0$ and a correct conclusion. E.g. (Bobby's) claim is sometimes true

Assume that $n^{2}$ is even and $n$ is odd.
So $n=2 k+1$, where $k$ is an integer.
$n^{2}=(2 k+1)^{2}=4 k^{2}+4 k+1$ So $n^{2}$ is odd which contradicts $n^{2}$ is even.
So (Elsa's) claim is true.
A1
(2)

For an explanation or statement to show when (Ying's) claim "the sum of two different irrational numbers is irrational" fails
This could be e.g.

- $\pi, 9-\pi$; sum $=\pi+9-\pi=9$ is not irrational

Followed by an explanation or statement to show when (Ying's) claim "the sum of two different irrational numbers is irrational" is true.
This could be e.g.

- $\pi, 9+\pi$; sum $=\pi+9+\pi=2 \pi+9$ is irrational
and a correct conclusion. E.g.
- (Ying's) claim is sometimes true


## Question 8 Notes:

| (i) |  |
| :---: | :---: |
| M1: | Attempts to <br> - complete the square or <br> - find the minimum by differentiation or <br> - draw a graph of $\mathrm{f}(y)=y^{2}-4 y+7$ |
| A1: | Completes the proof by showing $y^{2}-4 y+7$ is positive for all real values of $y$ with no errors seen in their working. |
| (ii) |  |
| M1: | See scheme |
| A1: | See scheme |
| (ii) |  |
| Alt 1 |  |
| M1: | Assumes $\mathrm{e}^{3 x} \geqslant \mathrm{e}^{2 x}$, takes logarithms and rearranges to make $x$ the subject of their inequality |
| A1: | See scheme |
| (iii) |  |
| M1: | Begins the proof by negating Elsa's claim and attempts to define $n$ as an odd number |
| A1: | Shows $n^{2}=4 k^{2}+4 k+1$, where $n$ is correctly defined and gives a correct conclusion |
| (iv) |  |
| M1: | See scheme |
| A1: | See scheme |

