

Question	Scheme	Marks	AOs
10	$V = 4\pi h(h + 6) = 4\pi h^2 + 24\pi h \quad 0 \leq h \leq 25; \frac{dV}{dt} = 80\pi$		
(a)	Time = $\frac{4\pi(24)(24 + 6)}{80\pi} = \frac{2880\pi}{80\pi} = 36$ (s) *	B1 *	3.4
		(1)	
(b)	When $t = 8, V = 80\pi(8) = 640\pi \Rightarrow 640\pi = 4\pi h(h + 6)$	M1	3.1a
	$160 = h(h + 6) \Rightarrow h^2 + 6h - 160 = 0 \Rightarrow (h + 16)(h - 10) = 0 \Rightarrow h = \dots$	M1	1.1b
	$\{h = -16, \text{reject}\}, h = 10$	A1	1.1b
	$\frac{dV}{dh} = 8\pi h + 24\pi$	M1	1.1b
		A1	1.1b
	$\left\{ \frac{dV}{dh} \times \frac{dh}{dt} = \frac{dV}{dt} \Rightarrow \right\} (8\pi h + 24\pi) \frac{dh}{dt} = 80\pi$	M1	3.1a
	When $h = 10, \left\{ \frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh} = \right\} \frac{80\pi}{(8\pi(10) + 24\pi)} \left\{ = \frac{80\pi}{124\pi} \right\}$	M1	3.4
	When $h = 10, \frac{dh}{dt} = \frac{10}{13}$ (cm s ⁻¹) or awrt 0.769 (cm s ⁻¹)	A1	1.1b
		(8)	

(9 marks)

Question 10 Notes:

(a)	
B1*:	Uses the model to show that it takes 36 seconds to fill the bowl from empty to a height of 24 cm
(b)	
M1:	Complete strategy to find the value of h when $t = 8$
M1:	Uses $\frac{dV}{dt} = 80\pi$ to deduce the volume of water in the bowl, V , after 8 seconds and sets this result to $4\pi h(h + 6)$
A1:	Finds $h = 10$
M1:	Differentiates V with respect to h to give $\pm\alpha h \pm\beta; \alpha, \beta \neq 0$
A1:	$8\pi h + 24\pi$
M1:	A complete strategy of forming an equation relating $\frac{dh}{dt}$ to 80π
	E.g. applies $\left(\text{their } \frac{dV}{dh} \right) \times \frac{dh}{dt} = 80\pi$
M1:	Substitutes their $h = "10"$ into their model for $\frac{dh}{dt}$ which is in the form $\frac{80\pi}{\left(\text{their } \frac{dV}{dh} \right)}$,
	where their h has been found from solving a quadratic equation in h
A1:	$\frac{10}{13}$ or awrt 0.769