

Question	Scheme	Marks	AOs
11 (i)	$\{y = a^x \Rightarrow\} \ln y = \ln a^x \Rightarrow \ln y = x \ln a \Rightarrow \frac{1}{y} \frac{dy}{dx} = \ln a$	M1	2.1
	$\frac{dy}{dx} = y \ln a \Rightarrow \frac{dy}{dx} = a^x \ln a *$	A1*	1.1b
		(2)	
(i) Alt 1	$\{y = a^x \Rightarrow\} y = e^{x \ln a} \Rightarrow \frac{dy}{dx} = (\ln a) e^{x \ln a}$	M1	2.1
	$\Rightarrow \frac{dy}{dx} = a^x \ln a *$	A1*	1.1b
		(2)	
(ii)	$\frac{d}{dy}(2 \tan y) = 2 \sec^2 y$	M1	1.1b
	$\{x = 2 \tan y \Rightarrow\} \frac{dx}{dy} = 2 \sec^2 y \quad \text{or} \quad 1 = (2 \sec^2 y) \frac{dy}{dx}$	A1	1.1b
	$\frac{dx}{dy} = 2(1 + \tan^2 y) \quad \text{or} \quad 1 = 2(1 + \tan^2 y) \frac{dy}{dx}$	M1	1.1b
	E.g. $\frac{dx}{dy} = 2 \left(1 + \left(\frac{x}{2} \right)^2 \right) \Rightarrow \frac{dx}{dy} = 2 \left(1 + \frac{x^2}{4} \right) \Rightarrow \frac{dx}{dy} = 2 + \frac{x^2}{2}$ $\Rightarrow \frac{dx}{dy} = \frac{4 + x^2}{2} \Rightarrow \frac{dy}{dx} = \frac{2}{4 + x^2}$	A1	2.1
		(4)	
(ii) Alt 1	$\{x = 2 \tan y \Rightarrow\} y = \arctan \left(\frac{x}{2} \right) \Rightarrow \frac{dy}{dx} = \frac{1}{\left(1 + \left(\frac{x}{2} \right)^2 \right)} \times \left(\frac{1}{2} \right)$	M1	1.1b
		M1	1.1b
		A1	1.1b
	$\Rightarrow \frac{dy}{dx} = \frac{1}{2 \left(1 + \frac{x^2}{4} \right)} \Rightarrow \frac{dy}{dx} = \frac{1}{\left(2 + \frac{x^2}{2} \right)} \Rightarrow \frac{dy}{dx} = \frac{1}{\left(\frac{4 + x^2}{2} \right)}$ $\Rightarrow \frac{dy}{dx} = \frac{2}{4 + x^2}$	A1	2.1
		(4)	
(6 marks)			

Question 11 Notes:**(i)**

M1: Applies the natural logarithm to both sides of $y = a^x$, applies the power law of logarithms and applies implicit differentiation to the result

A1*: Shows $\frac{dy}{dx} = a^x \ln a$, with no errors seen

(i)**Alt 1**

M1: Rewrites $y = a^x$ as $y = e^{x \ln a}$ and writes $\frac{dy}{dx} = c e^{x \ln a}$, where c can be 1

A1*: Shows $\frac{dy}{dx} = a^x \ln a$, with no errors seen

(ii)

M1: Evidence of $2 \tan y$ being differentiated to $2 \sec^2 y$

A1: Differentiates correctly to show that $x = 2 \tan y$ gives $\frac{dx}{dy} = 2 \sec^2 y$ or $1 = (2 \sec^2 y) \frac{dy}{dx}$

M1: Applies $\sec^2 y = 1 + \tan^2 y$ to their differentiated expression

A1: Shows that $\frac{dy}{dx} = \frac{2}{4 + x^2}$, with no errors seen

(ii)**Alt 1**

M1: Evidence of $\arctan(\lambda x)$; $\lambda \neq 0$ being differentiated to $\lambda \left(\frac{1}{1 + (\mu x^2)} \right)$; $\lambda, \mu \neq 0$

Note: λ can be 1 for this mark

M1: Differentiates $y = \arctan(\lambda x)$; $\lambda \neq 0$, $\lambda \neq 1$ to give an expression of the form $\frac{1}{(1 + (\lambda x)^2)} \times (\lambda)$

A1: Differentiates $y = \arctan\left(\frac{x}{2}\right)$ correctly to give $\frac{dy}{dx} = \frac{1}{\left(1 + \left(\frac{x}{2}\right)^2\right)} \times \left(\frac{1}{2}\right)$, o.e.

A1: Shows that $\frac{dy}{dx} = \frac{2}{4 + x^2}$, with no errors seen