Question 10

General points for marking question 10 (i):

- Students who just try random numbers in part (i) are not going to score any marks.
- Students can mix and match methods. Eg you may see odd numbers via logic and even via algebra
- Students who state $4m^2+2$ **cannot be divided** by (instead of is not divisible by) cannot be awarded credit for the accuracy/explanation marks, unless they state correctly that $4m^2+2$ **cannot be divided by 4 to give an integer.**
- Students who write $n^2+2=4k \Rightarrow k=\frac{1}{4}n^2+\frac{1}{2}$ which is not a whole number gains no credit unless they then start to look at odd and even numbers for instance
- Proofs via induction usually tend to go nowhere unless they proceed as in the main scheme
- Watch for unusual methods that are worthy of credit (See below)
- If the final conclusion is $n\in\mathbb{R}$ then the final mark is withheld. $n\in\mathbb{Z}^+$ is correct

Watch for methods that may not be in the scheme that you feel may deserve credit.

If you are uncertain of a method please refer these up to your team leader.

Eg 1. Solving part (i) by modulo arithmetic.

All $n\in\mathbb{N}$ mod 4	0	1	2	3
All $n^2\in\mathbb{N}$ mod 4	0	1	0	1
All $n^2+2\in\mathbb{N}$ mod 4	2	3	2	3

Hence for all n, n^2+2 is not divisible by 4.

Question 10 (i)	Scheme	Marks	AOs

Notes: Note that M0 A0 M1 A1 and M0 A0 M1 A0 are not possible due to the way the scheme is set up

(i)

M1: Awarded for setting up the proof for either the even or odd numbers.

A1: Concludes correctly with a reason why n^2+2 cannot be divisible by 4 for either n odd or even.

dM1: Awarded for setting up the proof for both even and odd numbers

A1: Fully correct proof with valid explanation and conclusion for all n

Example of an algebraic proof

For $n=2m$, $n^2+2=4m^2+2$	M1	2.1
Concludes that this number is not divisible by 4 (as the explanation is trivial)	A1	1.1b
For $n=2m+1$, $n^2+2=(2m+1)^2+2=...$ FYI $(4m^2+4m+3)$	dM1	2.1
Correct working and concludes that this is a number in the 4 times table add 3 so cannot be divisible by 4 or writes $4(m^2+m)+3$ ....AND states ......hence true for all	A1*	2.4
	(4)	

Example of a very similar algebraic proof

For $n = 2m,\quad \dfrac{4m^2+2}{4} = m^2 + \dfrac{1}{2}$	M1	2.1
Concludes that this is not divisible by 4 due to the $\dfrac{1}{2}$ (A suitable reason is required)	A1	1.1b
For $n = 2m+1,\quad \dfrac{n^2+2}{4} = \dfrac{4m^2+4m+3}{4} = m^2+m+\dfrac{3}{4}$	dM1	2.1
Concludes that this is not divisible by 4 due to the $\dfrac{3}{4}$ ...AND states ...... hence for all n, n^2+2 is not divisible by 4	A1*	2.4
	(4)	

Example of a proof via logic

When n is odd, "odd $\times$odd" = odd	M1	2.1
so n^2+2 is odd , so (when n is odd) n^2+2 cannot be divisible by 4	A1	1.1b
When n is even, it is a multiple of 2, so "even $\times$even" is a multiple of 4	dM1	2.1
Concludes that when n is even n^2+2 cannot be divisible by 4 because n^2 is divisible by 4…..AND STATES ……..trues for all n.	A1*	2.4
	(4)	

Example of proof via contradiction

Sets up the contradiction ‘Assume that n^2+2 is divisible by 4 $\Rightarrow n^2+2=4k$ ’	M1	2.1
$\Rightarrow n^2 = 4k-2 = 2(2k-1)$ and concludes even Note that the M mark (for setting up the contradiction must have been awarded)	A1	1.1b
States that n^2 is even, then n is even and hence n^2 is a multiple of 4	dM1	2.1
Explains that if n^2 is a multiple of 4 then n^2+2 cannot be a multiple of 4 and hence divisible by 4 Hence there is a contradiction and concludes Hence true for all n.	A1*	2.4
	(4)	

A similar proof exists via contradiction where

A1: $n^2 = 2(2k-1) \Rightarrow n = \sqrt{2}\times\sqrt{2k-1}$

dM1: States that $2k-1$ is odd, so does not have a factor of 2, meaning that n is irrational

Question 10 (ii)	Scheme	Marks	AOs

(ii)

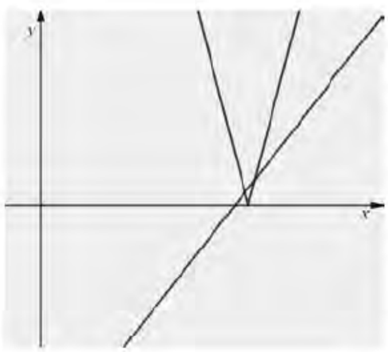
M1: States or implies 'sometimes true' or 'not always true' and gives an example where it is not true.

A1: and gives an example where it is true,

Proof using numerical values

SOMETIMES TRUE and chooses any number $x: 9.25 < x < 9.5$ and shows false Eg $x = 9.4$ $\|3x-28\| = 0.2$ and $x - 9 = 0.4$ ×	M1	2.3
Then chooses a number where it is true Eg $x = 12$ $\|3x-28\| = 8$ $x - 9 = 3$ ✓	A1	2.4
	(2)	

Graphical Proof



States or implies "sometimes true" Sketches both graphs on the same axes. Expect shapes and relative positions to be correct. V shape on +ve x -axis Linear graph with +ve gradient intersecting twice	M1	2.3
Graphs accurate and explains that as there are points where $\|3x-28\| < x-9$ and points where $\|3x-28\| > x-9$ oe in words like 'above' and 'below' or 'dips below at one point'	A1	2.4
	(2)	

Proof via algebra

States sometimes true and attempts to solve both $3x-28 < x-9$ and $-3x+28 < x-9$ or one of these with the bound $9.\dot{3}$	M1	2.3
States that it is false when $9.25 < x < 9.5$ or $9.25 < x < 9.\dot{3}$ or $9.\dot{3} < x < 9.5$	A1	2.4
	(2)	

Alt: It is possible to find where it is always true

States sometimes true and attempts to solve where it is just true Solves both $3x-28 \geqslant x-9$ and $-3x+28 \geqslant x-9$	M1	2.3
States that it is false when $9.25 < x < 9.5$ or $9.25 < x < 9.\dot{3}$ or $9.\dot{3} < x < 9.5$	A1	2.4
	(2)	