| Question | Scheme | Marks | AOs |
|-----------|--|-------|------|
| 14 (a) | Attempts to differentiate $x = 4 \sin 2y$ and inverts $\frac{dx}{dy} = 8 \cos 2y \Longrightarrow \frac{dy}{dx} = \frac{1}{8 \cos 2y}$ | M1 | 1.1b |
| | $\operatorname{At}(0,0) \ \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{8}$ | A1 | 1.1b |
| | | (2) | |
| (b) | (i) Uses $\sin 2y \approx 2y$ when y is small to obtain $x \approx 8y$ | B1 | 1.1b |
| | (ii) The value found in (a) is the gradient of the line found in (b)(i) | B1 | 2.4 |
| | | (2) | |
| (c) | Uses their $\frac{dy}{dx}$ as a function of y and, using both $\sin^2 2y + \cos^2 2y = 1$ and $x = 4\sin 2y$ in an attempt to write $\frac{dy}{dx}$ or $\frac{dx}{dy}$ as a function of x Allow for $\frac{dy}{dx} = k \frac{1}{\cos 2y} = \frac{1}{\sqrt{1 - (x)^2}}$ | M1 | 2.1 |
| | A correct answer $\frac{dy}{dx} = \frac{1}{8\sqrt{1-\left(\frac{x}{4}\right)^2}}$ or $\frac{dx}{dy} = 8\sqrt{1-\left(\frac{x}{4}\right)^2}$ | Al | 1.1b |
| | and in the correct form $\frac{dy}{dx} = \frac{1}{2\sqrt{16 - x^2}}$ | A1 | 1.1b |
| | | (3) | |
| (7 marks) | | | |

(a)

M1: Attempts to differentiate $x = 4 \sin 2y$ and inverts.

Allow for
$$\frac{dx}{dy} = k \cos 2y \Rightarrow \frac{dy}{dx} = \frac{1}{k \cos 2y}$$
 or $1 = k \cos 2y \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{k \cos 2y}$
Alternatively, changes the subject and differentiates $x = 4 \sin 2y \Rightarrow y = \dots \arcsin\left(\frac{x}{4}\right) \Rightarrow \frac{dy}{dx} = \frac{\dots}{\sqrt{1 - \left(\frac{x}{4}\right)^2}}$

It is possible to approach this from $x = 8 \sin y \cos y \Rightarrow \frac{dx}{dy} = \pm 8 \sin^2 y \pm 8 \cos^2 y$ before inverting

A1: $\frac{dy}{dx} = \frac{1}{8}$ Allow both marks for sight of this answer as long as no incorrect working is seen (See below)

Watch for candidates who reach this answer via $\frac{dx}{dy} = 8\cos 2x \Rightarrow \frac{dy}{dx} = \frac{1}{8\cos 2x}$ This is M0 A0

(b)(i)

B1: Uses $\sin 2y \approx 2y$ when y is small to obtain x = 8y or such as x = 4(2y).

Do not allow $\sin 2y \approx 2\theta$ to get $x = 8\theta$ but allow recovery in (b)(i) or (b)(ii)

Double angle formula is B0 as it does not satisfy the demands of the question.

(b)(ii)

B1: Explains the relationship between the answers to (a) and (b) (i).

For this to be scored the first three marks, in almost all cases, must have been awarded and the statement must refer to both answers

Allow for example "The gradients are the same $\left(=\frac{1}{8}\right)$ " 'both have $m = \frac{1}{8}$ '

Do not accept the statement that 8 and $\frac{1}{8}$ are reciprocals of each other unless further correct work explains the relationship in terms of $\frac{dx}{dx}$ and $\frac{dy}{dx}$

(c)

M1: Uses their $\frac{dy}{dx}$ as a function of y and, using both $\sin^2 2y + \cos^2 2y = 1$ and $x = 4\sin 2y$, attempts to

write $\frac{dy}{dx}$ or $\frac{dx}{dy}$ as a function of x. The $\frac{dy}{dx}$ may not be seen and may be implied by their calculation.

A1: A correct (un-simplified) answer for
$$\frac{dy}{dx}$$
 or $\frac{dx}{dy}$ Eg. $\frac{dy}{dx} = \frac{1}{8\sqrt{1-\left(\frac{x}{4}\right)^2}}$

A1: $\frac{dy}{dx} = \frac{1}{2\sqrt{16-x^2}}$ The $\frac{dy}{dx}$ must be seen at least once in part (c) of this solution

.....

Alt to (c) using arcsin

M1: Alternatively, changes the subject and differentiates

$$x = 4\sin 2y \rightarrow y = \dots \arcsin\left(\frac{x}{4}\right) \rightarrow \frac{dy}{dx} = \frac{\dots}{\sqrt{1 - \left(\frac{x}{4}\right)^2}}$$

Condone a lack of bracketing on the
$$\frac{x}{4}$$
 which may appear as $\frac{x^2}{4}$

A1:
$$\frac{dy}{dx} = \frac{\frac{1}{8}}{\sqrt{1 - \left(\frac{x}{4}\right)^2}} \text{ oe}$$
A1:
$$\frac{dy}{dx} = \frac{1}{2\sqrt{16 - x^2}}$$