

Question	Scheme	Marks	AOs
13 (a)	Uses $V = \frac{1}{3}\pi r^2 h$ with $\frac{r}{h} = \frac{2.5}{4}$ to establish $V = f(h^3)$ and differentiates	M1	2.1
	$\frac{dV}{dh} = \frac{75}{192}\pi h^2$	A1	1.1b
	States or uses $\frac{dV}{dt} = -\frac{\pi}{512}\sqrt{h}$	B1	1.1b
	Uses $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$ with their $\frac{dV}{dt}$ and their $\frac{dV}{dh}$	M1	3.1b
	$--\frac{\pi}{512}\sqrt{h} = \frac{75}{192}\pi h^2 \times \frac{dh}{dt} \Rightarrow h^{\frac{3}{2}} \frac{dh}{dt} = -\frac{1}{200} \quad *$	A1*	2.1
		(5)	
(b)	$\int h^{\frac{3}{2}} dh = -\int \frac{1}{200} dt \Rightarrow \frac{2}{5} h^{\frac{5}{2}} = -\frac{1}{200} t + c$	M1	1.1b
	Substitutes $t = 0, h = 4 \Rightarrow c = \left(\frac{64}{5}\right)$	dM1	3.4
	$\frac{2}{5} h^{\frac{5}{2}} = -\frac{1}{200} t + \frac{64}{5} \text{ oe}$	A1	3.3
		(3)	
(c)	Substitutes $h = 0 \Rightarrow 0 = -\frac{1}{200} t + \frac{64}{5} \Rightarrow t = \dots$	M1	3.4
	$t = 2560 \text{ seconds} = 42 \text{ minutes } 40 \text{ seconds}$	A1	3.2a
	States that the "real" time and the "predicted" times are very close so model seems suitable	A1	3.5a
		(3)	

(11 marks)

Notes:

(a)

M1: Uses $V = \frac{1}{3}\pi r^2 h$ with $\frac{r}{h} = \frac{2.5}{4}$ or equivalent to establish V as a function of h^3 which is then differentiated to an expression in h^2

A1: $\frac{dV}{dh} = \frac{75}{192}\pi h^2$

B1: Uses the information given in the question to states or uses $\frac{dV}{dt} = -\frac{\pi}{512}\sqrt{h}$

M1: Uses $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$ with their $\frac{dV}{dt}$ and their $\frac{dV}{dh}$ to form an equation linking $\frac{dh}{dt}$ and h

A1*: Proceeds correctly to the given equation $h^{\frac{3}{2}} \frac{dh}{dt} = -\frac{1}{200}$

(b)

M1: Integrates both sides to $ah^{\frac{5}{2}} = bt + c$. Condone the omission of $+c$ for this mark

dM1: Uses the model to find c

A1: Finds the equation of the model Eg. $\frac{2}{5}h^{\frac{5}{2}} = -\frac{1}{200}t + \frac{64}{5}$ or equivalent such as $h^{\frac{5}{2}} = -\frac{1}{80}t + 32$

(c)

M1: Uses the model with $h = 0$ and proceeds to find t

A1: Achieves $t = 2560$ seconds and converts this or the 43 minutes as required in order to test the model

A1: States that the "real" time and the "predicted" times are very close so the model seems suitable