Question	Scheme	Marks	AOs
14(a)	Uses $y = 3\sin 2t = 6\sin t\cos t$ and attempts to square	M1	2.1
	$y^2 = 9x^2\cos^2 t$	A1	1.1b
	Uses $\cos^2 t = 1 - \sin^2 t$ with $\sin t = \dfrac{x}{2}$ $$y^2 = 9x^2\left(1 - \frac{x^2}{4}\right)$$	M1	2.1
	$y^2 = \dfrac{9}{4}x^2\left(4 - x^2\right)$	A1	1.1b
		(4)	
(b)	Deduces that the radius of the circle is given by $$r^2 = x^2 + y^2$$	M1	3.1a
	$r^2 = x^2 + \dfrac{9}{4}x^2\left(4 - x^2\right)$	A1	1.1b
	Circle touches curve when r is a maximum so differentiate $r^2 = 10x^2 - \dfrac{9}{4}x^4 \Rightarrow 2r\dfrac{\mathrm{d}r}{\mathrm{d}x} = 20x - 9x^3$ and set $\dfrac{\mathrm{d}r}{\mathrm{d}x} = 0 \Rightarrow x = \sqrt{\dfrac{20}{9}}$	M1	3.1a
	Finds $r^2 = 10x^2 - \dfrac{9}{4}x^4$ with their $x = \sqrt{\dfrac{20}{9}}$	dM1	2.1
	$r = \dfrac{10}{3}$	A1	1.1b
		(5)	
			(9 marks)

(a)

M1: Uses $\sin 2t = 2\sin t\cos t$ and attempts to square $y = 3\sin 2t$

A1: $y^2 = 9x^2\cos^2 t$

M1: Uses $\cos^2 t = 1 - \sin^2 t$ with $\sin t = \dfrac{x}{2}$ in an attempt to get an equation in just *x* and *y*

A1: $y^2 = \dfrac{9}{4}x^2\left(4 - x^2\right)$

Note: It is possible to use the given equation $y^2 = kx^2\left(4 - x^2\right)$. The first M1 is scored for substituting in both *x* and *y* and using $\sin 2t = 2\sin t\cos t$. The second M1 is for using $4 - 4\sin^2 t = 4\cos^2 t$. For the A1 to be awarded there must be some minimal statement such as ✓ it can be expressed in this form

(b)

M1: For deducing that the radius of the circle can be found from $x^2 + y^2$

A1: A correct statement for r^2 in terms of one variable.

Look for $r^2 = x^2 + \frac{9}{4}x^2\left(4 - x^2\right)$ or $r^2 = 4\sin^2 t + 9\sin^2 2t$

M1: For a full method of finding a value of x or t where the circle touches the curve.

For this to be scored expect to see an attempt at implicit differentiation (or equivalent) followed by an attempt to find where $\frac{\mathrm{d}r}{\mathrm{d}x} = 0$.

Also scored for an attempt at completing the square $r^2 = 10x^2 - \frac{9}{4}x^4 = -\frac{9}{4}\left(x^2 - \frac{20}{9}\right)^2 + \frac{100}{9}$

M1: For showing all steps required to find r or r^2

A1: $r = \frac{10}{3}$

(b)	Deduces that the radius of the circle is given by $$r^2 = x^2 + y^2$$	M1	2.2a
	$$r^2 = 4\sin^2 t + 9\sin^2 2t$$	A1	1.1b
	Circle touches curve when r is a maximum so differentiate $\Rightarrow 2r\frac{\mathrm{d}r}{\mathrm{d}t} = 8\sin t\cos t + 36\sin 2t\cos 2t$ And set $\frac{\mathrm{d}r}{\mathrm{d}t} = 0 \Rightarrow 0 = 4\times\cancel{2\sin t\cos t} + 36\cancel{\sin 2t}\cos 2t$	M1	3.1a
	Finds $r^2 = 4\sin^2 t + 9\sin^2 2t$ with their $\cos 2t = -\frac{1}{9}$	dM1	2.1
	$\cos 2t = -\frac{1}{9} \Rightarrow \sin^2 t = \frac{5}{9}$ and $r^2 = 4\times\frac{5}{9} + 9\left(1 - \frac{1}{81}\right) = \frac{100}{9}$	A1	1.1b
		(5)	

Alt via simultaneous equations:

M1: Solves $r^2 = x^2 + y^2$ and $y^2 = \frac{9}{4}x^2\left(4 - x^2\right)$ to get an equation in either x or y.

A1: Either $9x^4 - 40x^2 + 4r^2 = 0$ or $9y^4 + \left(40 - 18r^2\right)y^2 + 9r^4 - 36r^2 = 0$

M1: Attempts $b^2 - 4ac = 0$ for their quartic equation of a form $ax^4 + bx^2 + c = 0$ where either *a, b* or *c* are dependent upon *r*

dM1: Uses $b^2 - 4ac = 0$ for their quartic equation of a form $ax^4 + bx^2 + c = 0$ where either *a, b* or *c* are dependent upon r to find a value for *r*

A1: $r = \frac{10}{3}$