

Question	Scheme	Marks	AOs
10 (a)	$x = u^2 + 1 \Rightarrow dx = 2udu$ oe	B1	1.1b
	Full substitution $\int \frac{3dx}{(x-1)(3+2\sqrt{x-1})} = \int \frac{3 \times 2u du}{(u^2+1-1)(3+2u)}$	M1	1.1b
	Finds correct limits e.g. $p = 2, q = 3$	B1	1.1b
	$= \int \frac{3 \times 2 \cancel{u} du}{u^{\cancel{2}}(3+2u)} = \int \frac{6 du}{u(3+2u)}$ *	A1*	2.1
		(4)	
(b)	$\frac{6}{u(3+2u)} = \frac{A}{u} + \frac{B}{3+2u} \Rightarrow A = \dots, B = \dots$	M1	1.1b
	Correct PF. $\frac{6}{u(3+2u)} = \frac{2}{u} - \frac{4}{3+2u}$	A1	1.1b
	$\int \frac{6 du}{u(3+2u)} = 2\ln u - 2\ln(3+2u) \quad (+c)$	dM1 A1ft	3.1a 1.1b
	Uses limits $u = "3", u = "2"$ with some correct ln work leading to $k \ln b$ E.g. $(2\ln 3 - 2\ln 9) - (2\ln 2 - 2\ln 7) = 2\ln \frac{7}{6}$	M1	1.1b
	$\ln \frac{49}{36}$	A1	2.1
	(6)		
(10 marks)			
Notes: Mark (a) and (b) together as one complete question			

(a)

B1: $dx = 2udu$ or exact equivalent. E.g. $\frac{dx}{du} = 2u, \frac{du}{dx} = \frac{1}{2}(x-1)^{-\frac{1}{2}}$

M1: Attempts a full substitution of $x = u^2 + 1$, including $dx \rightarrow \dots udu$ to form an integrand in terms of u .
Condone slips but there should be an attempt to use the correct substitution on the denominator.

B1: Finds correct limits either states $p = 2, q = 3$ or sight of embedded values as limits to the integral

A1*: Clear reasoning including one fully correct intermediate line, including the integral signs, leading to the given expression ignoring limits. So B1, M1, B0, A1 is possible if the limits are incorrect, omitted or left as 5 and 10.

(b)

M1: Uses correct form of PF leading to values of A and B .

A1: Correct PF $\frac{6}{u(3+2u)} = \frac{2}{u} - \frac{4}{3+2u}$ (Not scored for just the correct values of A and B)

dM1: This is an overall problem solving mark. It is for using the correct PF form and integrating using lns.
Look for $P \ln u + Q \ln(3+2u)$

A1ft: Correct integration for their $\frac{A}{u} + \frac{B}{3+2u} \rightarrow A \ln u + \frac{B}{2} \ln(3+2u)$ with or without modulus signs

M1: Uses their 2 and 3 as limits, with at least one correct application of the addition law or subtraction law leading to the form $k \ln b$ or $\ln a$. PF's must have been attempted. Condone bracketing slips. Alternatively changing the u 's back to x 's and use limits of 5 and 10.

A1: Proceeds to $\ln \frac{49}{36}$. Answers without working please send to review.