

Question	Scheme	Marks	AOs
13 (a)	Uses the sequence formula $a_{n+1} = \frac{k(a_n + 2)}{a_n}$ once with $a_1 = 2$	M1	1.1b
	$(a_1 = 2), a_2 = 2k, a_3 = k + 1, a_4 = \frac{k(k + 3)}{k + 1}$ Finds four consecutive terms and sets a_4 equal to a_1 (oe)	M1	3.1a
	$\frac{k(k + 3)}{k + 1} = 2 \Rightarrow k^2 + 3k = 2k + 2 \Rightarrow k^2 + k - 2 = 0 \quad *$	A1*	2.1
		(3)	
(b)	States that when $k = 1$, all terms are the same and concludes that the sequence does not have a period of order 3	B1	2.3
		(1)	
(c)	Deduces the repeating terms are $a_{1/4} = 2, a_{2/5} = -4, a_{3/6} = -1,$	B1	2.2a
	$\sum_{n=1}^{80} a_k = 26 \times (2 + -4 + -1) + 2 + -4$	M1	3.1a
	$= -80$	A1	1.1b
		(3)	

(7 marks)

Notes:

(a)

M1: Applies the sequence formula $a_{n+1} = \frac{k(a_n + 2)}{a_n}$ seen once.

This is usually scored in attempting to find the second term. E.g. for $a_2 = 2k$ or $a_{1+1} = \frac{k(2+2)}{2}$

M1: Attempts to find $a_1 \rightarrow a_4$ and sets $a_1 = a_4$. Condone slips.

Other methods are available. E.g. Set $a_4 = 2$, work backwards to find a_3 and equate to $k + 1$

There is no requirement to see either a_1 or any of the labels. Look for the correct terms in the correct order.

There is no requirement for the terms to be simplified

FYI $a_1 = 2, a_2 = 2k, a_3 = k + 1, a_4 = \frac{k(k + 3)}{k + 1}$ and so $2 = \frac{k(k + 3)}{k + 1}$

A1*: Proceeds to the given answer with accurate work showing all necessary lines. See MS for minimum

(b)

B1: States that when $k = 1$, all terms are the same and concludes that the sequence does not have a period of order 3.

Do not accept "the terms just repeat" or "it would mean all the terms of the sequence are 2"

There must be some reference to the fact that it does not have order 3. Accept it has order 1.

It is acceptable to state $a_2 = a_1 = 2$ and state that the sequence does not have order 3

(c)

B1: Deduces the repeating terms are $a_{1/4} = 2, a_{2/5} = -4, a_{3/6} = -1$,

M1: Uses a clear strategy to find the sum to 80 terms. This will usually be found using multiples of the first three terms.

For example you may see $\sum_{r=1}^{80} a_r = \left(\sum_{r=1}^{78} a_r \right) + a_{79} + a_{80} = 26 \times (2 + -4 + -1) + 2 + -4$

or $\sum_{r=1}^{80} a_r = \left(\sum_{r=1}^{81} a_r \right) - a_{81} = 27 \times (2 + -4 + -1) - (-1)$

For candidates who find in terms of k award for $27 \times 2 + 27 \times (2k) + 26 \times (k + 1)$ or $80k + 80$

If candidates proceed and substitute $k = -2$ into $80k + 80$ to get -80 then all 3 marks are scored.

A1: -80

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Note: Be aware that we have seen candidates who find the first three terms correctly, but then find

$26 \frac{2}{3} \times (2 + -4 + -1) = 26 \frac{2}{3} \times -3$ which gives the correct answer

but it is an incorrect method and should be scored B1 M0 A0

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