Question	Scheme	Marks	AOs
14 (a)	Uses the model to state $\frac{dV}{dt} = -c$ (for positive constant c)	B1	3.1b
	Uses $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$ with their $\frac{dV}{dt} = -c$ and $\frac{dV}{dr} = 4\pi r^2$	M1	2.1
	$-c = 4\pi r^2 \times \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = -\frac{c}{4\pi r^2} = -\frac{k}{r^2}$ *	A1*	2.2a
		(3)	
(b)	$\frac{dr}{dt} = -\frac{k}{r^2} \Rightarrow \int r^2 dr = \int -k dt$ and integrates with one side "correct"	M1	2.1
	$\frac{r^3}{3} = -kt(+\alpha)$	A1	1.1b
	Uses $t = 0, r = 40 \Rightarrow \alpha = ...$ $\alpha = \frac{64000}{3}$	M1	1.1b
	Uses $t = 5, r = 20$ & $\alpha = ... \Rightarrow k = ...$	M1	3.4
	$r^3 = 64000 - 11200t$ or exact equivalent	A1	3.3
		(5)	
(c)	Uses the equation of their model and proceeds to a limiting value for t E.g. "$64000 - 11200t$" ... $0 \Rightarrow t...$	M1	3.4
	For times up to and including $\frac{40}{7}$ seconds	A1ft	3.5b
		(2)	
			(10 marks)

Notes:

(a)

B1: Uses the model to state $\frac{dV}{dt} = -c$ (for positive constant c).

Any "letter" is acceptable here including k.

Note that $\frac{dV}{dt} = c$ is B0 unless they state that c is a negative constant.

M1: For an attempt to use $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$ with their $\frac{dV}{dt}$ and $\frac{dV}{dr} = 4\pi r^2$

Allow for an attempt to use $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$ with their $\frac{dV}{dt}$ and $\frac{dV}{dr} = \lambda r^2$ (Any constant is fine)

There is no requirement to use the correct formula for the volume of a sphere for this mark.

A1*: Proceeds to the given answer with an intermediate line equivalent to $\frac{dr}{dt} = -\frac{c}{4\pi r^2}$

If candidate started with $\frac{dV}{dt} = -k$ they must provide a minimal explanation how

$\frac{dr}{dt} = -\frac{k}{4\pi r^2} \to \frac{dr}{dt} = -\frac{k}{r^2}$. E.g $\frac{1}{4\pi}$ is a constant so replace $\frac{k}{4\pi}$ with k

It is not necessary to use the full formula for the volume of a sphere, eg allow $V = \kappa r^3$ but if it has been quoted it must be correct. So using $V = 4r^3$ can potentially score 2 of the 3 marks.

(b)

M1: For the key step of separating the variables correctly AND integrating one side with at least one index correct. The integral signs do not need to be seen.

A1: Correct integration E.g. $\frac{r^3}{3} = -kt(+\alpha)$ or equivalent. The $+\alpha$ is not required for this mark.

This may be awarded if k has been given a value.

M1: Uses the initial conditions to find a value for the constant of integration α

If a constant of integration is not present, or k has been given a pre defined value, then only the first two marks can be awarded in part (b)

The mark may be awarded if the equation has been adapted incorrectly. E.g. each term cube rooted.

M1: Uses the second set of conditions with their value of α to find k

This may be awarded if the equation has been adapted incorrectly. E.g. each term cube rooted.

A1: Obtains any correct equation for the model.

E.g. $r^3 = 64000 - 11200t$ or exact equivalent such as $\frac{r^3}{3} = \frac{64000}{3} - \frac{11200}{3}t$.

ISW after sight of a correct answer. Condone recurring decimals e.g. $21333.\dot{3}$ for $\frac{64000}{3}$

Do not award if **only the** rounded/truncated decimal equivalents to say $\frac{64000}{3}$ is used.

(c)

M1: Recognises that the model is only valid when $r \geqslant 0$ **and uses this to find *t*.** Condone $r > 0$

Award for an attempt to find the value of t when $r = 0$. See scheme.

It must be from an equation of the form $ar^n = b - ct, \quad a,b,c > 0$ which give + ve values of t.

A1ft: Allow valid for times up to (and including) $\frac{40}{7}$ seconds, 5.71 seconds. Allow $t < \frac{40}{7}$ or $t \leqslant \frac{40}{7}$

There is no requirement for the left hand side of the inequality, 0

States invalid for times greater than $\frac{40}{7}$ seconds, 5.71 seconds.

Follow through on their equation so allow $t <$ their "$\frac{64000}{11200}$" as long as this value is greater than 5

($t = 5$ is one of the values in the question)