Question	Scheme	Marks	AOs
15 (a)	$x^2\tan y = 9 \Rightarrow 2x\tan y + x^2\sec^2 y\dfrac{dy}{dx} = 0$	M1 A1	3.1a 1.1b
	Full method to get $\dfrac{dy}{dx}$ in terms of x using $\sec^2 y = 1+\tan^2 y = 1+f(x)$	M1	1.1b
	$\dfrac{dy}{dx} = \dfrac{-2x\times\dfrac{9}{x^2}}{x^2\left(1+\dfrac{81}{x^4}\right)} = \dfrac{-18x}{x^4+81}$ *	A1*	2.1
		(4)	
(b)	$\dfrac{dy}{dx} = \dfrac{-18x}{x^4+81}$ $\Rightarrow \dfrac{d^2y}{dx^2} = \dfrac{-18\times(x^4+81)-(-18x)(4x^3)}{(x^4+81)^2} = \dfrac{54(x^4-27)}{(x^4+81)^2}$ o.e.	M1 A1	1.1b 1.1b
	States that when $x<\sqrt[4]{27} \Rightarrow \dfrac{d^2y}{dx^2}<0$ when $x=\sqrt[4]{27} \Rightarrow \dfrac{d^2y}{dx^2}=0$ **AND** when $x>\sqrt[4]{27} \Rightarrow \dfrac{d^2y}{dx^2}>0$ giving a point of inflection when $x=\sqrt[4]{27}$	A1	2.4
		(3)	
			(7 marks)
Notes:			

(a)

M1: Attempts to differentiate $\tan y$ implicitly. Eg. $\tan y \to \sec^2 y\dfrac{dy}{dx}$ or $\cot y \to -\operatorname{cosec}^2 y\dfrac{dy}{dx}$

You may well see an attempt $\tan y = \dfrac{9}{x^2} \Rightarrow \sec^2 y\dfrac{dy}{dx} = ...$

When a candidate writes $x^2\tan y = 9 \Rightarrow x = 3\tan^{-\frac{1}{2}} y$ the mark is scored for $\tan^{-\frac{1}{2}} y \to ...\tan^{-\frac{3}{2}} y\sec^2 y$

A1: Correct differentiation $2x\tan y + x^2\sec^2 y\dfrac{dy}{dx} = 0$

Allow also $\sec^2 y\dfrac{dy}{dx} = -\dfrac{18}{x^3}$ or $2x = -9\operatorname{cosec}^2 y\dfrac{dy}{dx}$ amongst others

M1: Full method to get $\dfrac{dy}{dx}$ in terms of x using $\sec^2 y = 1+\tan^2 y = 1+f(x)$

A1*: Proceeds correctly to the given answer of $\dfrac{dy}{dx} = \dfrac{-18x}{x^4+81}$

(b)

M1: Attempts to differentiate the given expression using the product or quotient rule.

For example look for a correct attempt at $\frac{vu'-uv'}{v^2}$ with $u=-18x, v=x^4+81, u'=\pm18, v'=...x^3$

If no method is seen or implied award for $\frac{\pm18\times(x^4+81)\pm18x(ax^3)}{(x^4+81)^2}$

Using the product rule award for $\pm18(x^4+81)^{-1}\pm18x(x^4+81)^{-2}\times cx^3$

A1: Correct **simplified** $\frac{d^2y}{dx^2}=\frac{54(x^4-27)}{(x^4+81)^2}$ o.e. such as $\frac{d^2y}{dx^2}=\frac{54x^4-1458}{(x^4+81)^2}$

Alternatively score for showing that when a correct (unsimplified) $\frac{d^2y}{dx^2}=0\Rightarrow x^4=27\Rightarrow x=\sqrt[4]{27}$

Or for substituting $x=\sqrt[4]{27}$ into an unsimplified but correct $\frac{d^2y}{dx^2}$ and showing that it is 0

A1: Correct explanation with a minimal conclusion and correct second derivative.

See scheme.

It can be also be argued from $x^4<27$, $x^4=27$ and $x^4>27$ provided the conclusion states that the point of inflection is at $x=\sqrt[4]{27}$

Alternatively substitutes values of x either side of $\sqrt[4]{27}$ and at $\sqrt[4]{27}$, into $\frac{d^2y}{dx^2}$, finds all three values and makes a minimal conclusion.

A different method involves finding $\frac{d^3y}{dx^3}$ and showing that $\frac{d^3y}{dx^3}\neq0$ and $\frac{d^2y}{dx^2}=0$ when $x=\sqrt[4]{27}$

FYI $\frac{d^3y}{dx^3}=\frac{23328x^3}{(x^4+81)^3}=0.219$ when $x=\sqrt[4]{27}$

Alternative part (a) using arctan

M1: Sets $y=\arctan\frac{9}{x^2}\rightarrow\frac{dy}{dx}=\frac{1}{1+\left(\frac{9}{x^2}\right)^2}\times...$ where ... could be 1

A2: $y=\arctan\frac{9}{x^2}\rightarrow\frac{dy}{dx}=\frac{1}{1+\left(\frac{9}{x^2}\right)^2}\times-\frac{18}{x^3}$

A1*: $\frac{dy}{dx}=\frac{1}{1+\frac{81}{x^4}}\times-\frac{18}{x^3}=\frac{-18x}{x^4+1}$ showing correct intermediate step and no errors.