Question	Scheme	Marks	AOs
16	Sets up the contradiction and factorises: There are positive integers p and q such that $(2p+q)(2p-q)=25$	M1	2.1
	If true then $\begin{matrix} 2p+q=25 \\ 2p-q=1 \end{matrix}$ or $\begin{matrix} 2p+q=5 \\ 2p-q=5 \end{matrix}$ **Award for deducing either of the above statements**	M1	2.2a
	Solutions are $p=6.5, q=12$ or $p=2.5, q=0$ Award for one of these	A1	1.1b
	This is a contradiction as there are no integer solutions hence there are no positive integers p and q such that $4p^2-q^2=25$	A1	2.1
		(4)	
	(4 marks)		
Notes:			

M1: For the key step in setting up the contradiction and factorising

M1: For deducing that for p and q to be integers then either $\begin{matrix} 2p+q=25 \\ 2p-q=1 \end{matrix}$ or $\begin{matrix} 2p+q=5 \\ 2p-q=5 \end{matrix}$ must be true.

Award for deducing either of the above statements.

You can ignore any reference to $\begin{matrix} 2p+q=1 \\ 2p-q=25 \end{matrix}$ as this could not occur for positive p and q.

A1: For correctly solving one of the given statements,

For $\begin{matrix} 2p+q=25 \\ 2p-q=1 \end{matrix}$ candidates only really need to proceed as far as $p=6.5$ to show the contradiction.

For $\begin{matrix} 2p+q=5 \\ 2p-q=5 \end{matrix}$ candidates only really need to find either p or q to show the contradiction.

Alt for $\begin{matrix} 2p+q=5 \\ 2p-q=5 \end{matrix}$ candidates could state that $2p+q \neq 2p-q$ if p,q are positive integers.

A1: For a complete and rigorous argument with both possibilities and a correct conclusion.

Question	Scheme	Marks	AOs
16 Alt 1	Sets up the contradiction, attempts to make q^2 or $4p^2$ the subject and states that either $4p^2$ is even(*) , or that q^2 (or q) is odd (**) Either There are positive integers p and q such that $4p^2-q^2=25 \Rightarrow q^2=4p^2-25$ with * or ** Or There are positive integers p and q such that $4p^2-q^2=25 \Rightarrow 4p^2=q^2+25$ with * or **	M1	2.1
	Sets $q=2n\pm1$ and expands $(2n\pm1)^2=4p^2-25$	M1	2.2a
	Proceeds to an expression such as $4p^2=4n^2+4n+26=4(n^2+n+6)+2$ $4p^2=4n^2+4n+26=4(n^2+n)+\frac{13}{2}$ $p^2=n^2+n+\frac{13}{2}$	A1	1.1b
	States This is a contradiction as $4p^2$ must be a multiple of 4 Or p^2 must be an integer And concludes there are no positive integers p and q such that $4p^2-q^2=25$	A1	2.1
		(4)	

Alt 2

An approach using odd and even numbers is unlikely to score marks.

To make this consistent with the Alt method, score

M1: Set up the contradiction and start to consider one of the cases below where q is odd, $m \neq n$.

Solutions using the same variable will score no marks.

M1: Set up the contradiction and start to consider BOTH cases below where q is odd, $m \neq n$.

No requirement for evens

A1: Correct work and deduction for one of the two scenarios where q is odd

A1: Correct work and deductions for both scenarios where q is odd with a final conclusion

Options	Example of Calculation	Deduction
p (even) q (odd)	$4p^2-q^2=4\times(2m)^2-(2n+1)^2=16m^2-4n^2-4n-1$	One less than a multiple of 4 so cannot equal 25
p (odd) q (odd)	$4p^2-q^2=4\times(2m+1)^2-(2n+1)^2=16m^2+16m-4n^2-4n+3$	Three more than a multiple of 4 so cannot equal 25