

$$f(x) = \frac{50x^2 + 38x + 9}{(5x+2)^2(1-2x)} \quad x \neq -\frac{2}{5} \quad x \neq \frac{1}{2}$$

Given that $f(x)$ can be expressed in the form

$$\frac{A}{5x+2} + \frac{B}{(5x+2)^2} + \frac{C}{1-2x}$$

where A , B and C are constants

(a) (i) find the value of B and the value of C

$$\text{(a)(i)} \quad 50x^2 + 38x + 9 \equiv A(5x+2)(1-2x) + B(1-2x) + C(5x+2)^2$$

(ii) show that $A = 0$

$$\text{(a) contd} \quad \frac{50}{4} + 19 + 9 = \frac{91}{4}C \Rightarrow C = \frac{91}{2} \times \frac{4}{91} = 2 \quad (1 \text{ mark}) \quad (4)$$

(b) (i) Use binomial expansions to show that, in ascending powers of x

$$f(x) = p + qx + rx^2 + \dots \quad \text{(a) contd} \quad \text{when } x = -\frac{2}{5},$$

where p , q and r are simplified fractions to be found.

$$50\left(-\frac{2}{5}\right)^2 + 38\left(-\frac{2}{5}\right) + 9 = 0 + B(1-2\left(-\frac{2}{5}\right)) + 0$$

$$\Rightarrow 50\left(\frac{4}{25}\right) - 38\left(\frac{2}{5}\right) + 9 = \frac{9}{5}B$$

(ii) Find the range of values of x for which this expansion is valid.

$$\Rightarrow B = \frac{5}{9}\left(8 - \frac{76}{5} + 9\right) = 1 \quad (7)$$

(1 mark)

(a)(ii) When $x = 0$,

$$50(0)^2 + 38(0) + 9 = A(5(0)+2)(1-2(0)) + B(1-2(0)) + C(5(0)+2)^2$$

$$9 = 2A + B + 4C \quad (1 \text{ mark})$$

but $B=1$, $C=2$ (from (a)(i))

$$\text{so } 9 = 2A + 1 + 4(2) \Rightarrow 9 = 2A + 9$$

$$\Rightarrow A = 0 \quad (1 \text{ mark})$$

(b)(i) $f(x) = (5x+2)^{-2} + 2(1-2x)^{-1}$

$$= \left(2\left(1 + \frac{5}{2}x\right)\right)^{-2} + 2(1-2x)^{-1}$$

$$= 2^{-2}\left(1 + \frac{5}{2}x\right)^{-2} + 2(1-2x)^{-1} \quad (1 \text{ mark})$$

$$= \frac{1}{4}\left(1 - 2\left(\frac{5}{2}x\right) + \frac{(-2)(-3)}{2}\left(\frac{5}{2}x\right)^2 + \dots\right) + 2\left(1 - 1(-2x) + \frac{(-1)(-2)}{2}(-2x)^2 + \dots\right) \quad (2 \text{ marks})$$

higher powers of x than x^2 are not required

$$= \frac{1}{4}\left(1 - 5x + \frac{75}{4}x^2 + \dots\right) + 2(1 + 2x + 4x^2 + \dots) = \left(\frac{1}{4} + 2\right) + \left(4 - \frac{5}{4}\right)x + \left(\frac{75}{16} + 8\right)x^2 + \dots$$

$$= \frac{9}{4} + \frac{11}{4}x + \frac{203}{16}x^2 + \dots \quad (4 \text{ marks})$$

(c) $\left|\frac{5}{2}x\right| < 1 \Rightarrow |x| < \frac{2}{5}$

& $|-2x| < 1 \Rightarrow |x| < \frac{1}{2}$

$|x| < \frac{2}{5}$ is the more restrictive constraint, so expansion is valid for $|x| < \frac{2}{5}$ (1 mark)