

$$f(x) = 3x^3 - 7x^2 + 7x - 10$$

(a) Use the factor theorem to show that $(x - 2)$ is a factor of $f(x)$

(b) Find the values of the constants a, b and c such that

$$f(x) \equiv (x - 2)(ax^2 + bx + c)$$

(3)

(c) Using your answer to part (b) show that the equation $f(x) = 0$ has only one real root.

(2)

(a) by factor theorem, if $(x-2)$ is a factor, then $f(2) = 0$

$$\begin{aligned}
 f(2) &= 3(2)^3 - 7(2)^2 + 7(2) - 10 \\
 &= 24 - 28 + 14 - 10 \\
 &= 0, \text{ so } (x-2) \text{ is a factor of } f(x) \quad (2 \text{ marks})
 \end{aligned}$$

(b)

$$\begin{array}{r}
 3x^2 - x + 5 \\
 x-2 \overline{) 3x^3 - 7x^2 + 7x - 10} \\
 \underline{-(3x^3 - 6x^2)} \\
 -x^2 + 7x \\
 \underline{-(-x^2 + 2x)} \\
 5x - 10 \\
 \underline{-(5x - 10)} \\
 0
 \end{array} \quad (1 \text{ mark})$$

so, $f(x) = (x-2)(3x^2 - x + 5)$ (2 marks)
 $a=3 \quad b=-1 \quad c=5$

(c) $x = 2$ is a root of $f(x)$
roots of $3x^2 - x + 5 = 0$ would be other roots of $f(x)$

$$\begin{aligned}
 \text{Discriminant} &= b^2 - 4ac = (-1)^2 - 4(3)5 \quad (1 \text{ mark}) \\
 &= +1 - 60 \\
 &= -59
 \end{aligned}$$

Discriminant < 0 , so $3x^2 - x + 5 = 0$ has no real roots

so $x = 2$ is the only real root of $f(x)$ (1 mark)